Derived algebraic/differential geometry

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Lecturers: Artan Sheshmani and Dennis Borisov

1. **Basic setting of derived geometry**

   (a) Lecture 1: Model and ∞-categories,

   (b) Lecture 2: Grothendieck topologies and homotopy descent

   (c) Lecture 3: Derived Artin stacks,

   (d) Lecture 4: Cotangent complexes.

   The goal: To collect the minimum set of tools needed to do algebraic geometry in the derived context.

   Literature: The general framework is given in:

   (a) B.Toën, G.Vezzosi. *Homotopical algebraic geometry I: topos theory.* Advances in Mathematics 193 (2005)

   (b) B.Toën, G.Vezzosi. *Homotopical algebraic geometry II: geometric stacks and applications.* Memoirs of the AMS 902 (2008)

   A however a more accessible source would be:
   B.Toën. *Simplicial presheaves and derived algebraic geometry.* In *Simplicial methods for operads and algebraic geometry.* Birkhäuser (2010).

   To deal with infinitesimal geometry another useful source will be:
   J.P.Pridham. *Presenting higher stacks as simplicial schemes.* Advances in Mathematics 238 (2013)

   To cover the C^∞-side we might need to look:

   (a) O.Ben-Bassat, K.Kremnizer. *Non-Archimedean analytic geometry as relative algebraic geometry.* arXiv:1312.0338

(b) D.Borisov, K.Kremnizer. *Quasi-coherent sheaves in differential geometry.* arXiv:1707.01145 [math.DG]

(c) D.Borisov, J.Noel. *Simplicial approach to derived differential manifolds.* (2011) arXiv:1112.0033v1 [math.DG]

2. **Loop spaces and differential forms**

(a) Lecture 5: De Rham complexes and S^1-equivariant schemes (loop spaces),

(b) Lecture 6: Chern character,

(c) Lectures 7 and 8: Local structure of closed differential forms in the derived sense,

(d) Lecture 9: Cyclic homology.

The goal: This is the algebraic heart of the course – here we learn the homological techniques that are needed for shifted symplectic forms.

Literature: The building blocks are

(a) B.Toën, G.Vezzosi. *Algèbres simpliciales S^1-équivariantes, théories de de Rham et théorèmes HKR multiplicatifs.* Composition Mathematica 147/06 (2011)

(b) B.Toën, G.Vezzosi. *Caractères de Chern, traces ëquivariantes et géométries algébriques dérivée.* Selecta Mathemtica 21/2 (2014)

(c) D.Ben-Zvi, D.Nadler. *Loop spaces and connections.* J. of Topology 5 (2012)

culminating in

(a) T.Pantev, B.Toën, M.Vaquié, G.Vezzosi. *Shifted symplectic structures.* Publ. math. de l'IHÉS 117/1 (2013)

(b) J-L.Loday. *Cyclic homology.* Springer (1992)

3. **Shifted symplectic structures.**

(a) Lecture 10: Definition and existence results,

(b) Lectures 11 and 12: Lagrangians and Lagrangian fibrations,

(c) Lecture 13: Intersections of Lagrangians.

(d) Lecture 14 and 15: Examples and applications

Goal: To see applications of the algebraic techniques from above in the geometric context of the actual moduli spaces.

Literature:

(a) T.Pantev, B.Toën, M.Vaquié, G.Vezzosi. *Shifted symplectic structures.* Publ. math. de l'IHÉS 117/1 (2013)

(b) Ch.Brav, V.Bussi, D.Joyce. *A Darboux theorem for derived schemes with shifted symplectic structure.* J. of the AMS 910 (2018)

(c) D.Joyce, P.Safronov. *A Lagrangian Neighbourhood Theorem for shifted symplectic derived schemes.* arXiv 1506.04024 [math.AG]

(d) D.Borisov, D.Joyce. *Virtual fundamental classes for moduli spaces of sheaves on Calabi–Yau four-folds.* Geometry and Topology 21 (2017).

4. **Uhlenbeck–Yau construction and correspondence**

Lectures 16, 17, 18. Will be planned as we go along.

References

[1] O.Ben-Bassat, K.Kremnizer. *Non-Archimedean analytic geometry as relative algebraic geometry.* arXiv:1312.0338

[2] D.Borisov, K.Kremnizer. *Quasi-coherent sheaves in differential geometry.* arXiv:1707.01145 [math.DG]

[3] D.Borisov, D.Joyce. *Virtual fundamental classes for moduli spaces of sheaves on Calabi–Yau four-folds.* Geometry and Topology 21 (2017).

[4] D.Borisov, J.Noel. *Simplicial approach to derived differential manifolds.* (2011) arXiv:1112.0033v1 [math.DG]

[5] J-L.Loday. *Cyclic homology.* Springer (1992)

[6] Ch.Brav, V.Bussi, D.Joyce. *A Darboux theorem for derived schemes with shifted symplectic structure.* J. of the AMS 910 (2018)

[7] D.Joyce, P.Safronov. *A Lagrangian Neighbourhood Theorem for shifted symplectic derived schemes.* arXiv 1506.04024 [math.AG]

[8] T.Pantev, B.Toën, M.Vaquié, G.Vezzosi. *Shifted symplectic structures.* Publ. math. de l'IHÉS 117/1 (2013)

[9] J.P.Pridham. *Presenting higher stacks as simplicial schemes.* Advances in Mathematics 238 (2013)

[10] B.Toën, G.Vezzosi. *Homotopical algebraic geometry I: topos theory.* Advances in Mathematics 193 (2005)

[11] B.Toën, G.Vezzosi. *Homotopical algebraic geometry II: geometric stacks and applications.* Memoirs of the AMS 902 (2008)

[12] B.Toën. *Simplicial presheaves and derived algebraic geometry.* In *Simplicial methods for operads and algebraic geometry.* Birkhäuser (2010)

[13] B.Toën, G.Vezzosi. *Algèbres simpliciales S^1-équivariantes, théories de de Rham et théorèmes HKR multiplicatifs.* Composition Mathematica 147/06 (2011)