Amplituhedra, Cluster Algebras, and Positive Geometry

May 29–31, 2024
Harvard CMSA, 20 Garden Street, Cambridge MA 02138

Wednesday, May 29, 2024

8:30 – 9:00 am	Registration and Breakfast
9:00 – 10:00 am	Jaroslav Trnka, UC Davis

Amplituhedron

I will review basics of the Amplituhedron, connection to the positive Grassmannian on the mathematical side, and the scattering amplitudes on the physics side.

10:00 – 10:15 am	Coffee Break
10:15 – 11:15 am	Khrystyna Serhiyenko, University of Kentucky (via Zoom)

Introduction to Cluster Algebras

Cluster algebras is a class of commutative rings with an intricate combinatorial structure. They were introduced by Fomin and Zelevinsky in 2002 to study total positivity and canonical basis in Lie Theory, but quickly evolved into a highly active research area with surprising connections to numerous other areas of mathematics and physics. In this course we will introduce cluster algebras and discuss their basic properties including positivity and Laurent phenomenon. We will also review cluster structures coming from coordinate rings of Grassmannians and the combinatorics of plabic graphs.

11:15 – 11:30 am	Coffee Break
11:30 – 12:30 pm	Thomas Lam, University of Michigan

Introductory Lecture on Positive Geometries

Positive geometries are semialgebraic spaces that appear in the study of scattering amplitudes. Examples include polytopes, totally nonnegative parts of flag varieties, and conjecturally, the amplituhedron. We will give a broad introduction to positive geometries, and to their canonical forms.

12:30 – 2:00 pm	Lunch Break
2:00 – 2:50 pm	Anastasia Volovich, Brown University

Scattering Amplitudes and Cluster Algebras

I will review some of the deep connections between cluster algebras and the (loop level) scattering amplitudes in N=4 super Yang-Mills theory, focusing on the cases of n=6 and 7 particles where the corresponding Grassmannian cluster algebras Gr(4,n) are finite and certain features of the amplitudes are known or believed to be true to all loop order.

2:50 – 3:00 pm	Coffee Break
3:00 – 3:50 pm	Marcus Spradlin, Brown University

Scattering Amplitudes, Positive Geometry and the Amplituhedron

I will review the status of (loop level) scattering amplitudes in N=4 super Yang-Mills theory for n>7, where the corresponding Grassmannian cluster algebras Gr(4,n) are infinite and novel features emerge, notably the appearance of certain "marginally positive" algebraic functions of cluster variables.

3:50 – 4:00 pm	Coffee Break
4:00 – 4:30 pm	Carolina Figueiredo, Princeton University

All-order splits and multi-soft limits for particle and string amplitude

The most important aspects of scattering amplitudes have long been thought to be associated with their poles. Recently a very different sort of "split" factorizations for a wide range of particle and string tree amplitudes have been discovered away from poles. In this talk, I will explain how natural properties of the binary geometry of the curve integral formulation for scattering amplitudes for Tr(Φ^3) theory give a simple, conceptual origin for these splits, that generalizes them to all

orders in the topological expansion. I will also explain how the splits allow us to access and compute loop-integrated multi-soft limits for particle and string amplitudes in Tr(Φ^3) theory, the non-linear sigma model and Yang-Mills theory.

4:30 – 5:15 pm	Yelena Mandelshtam, UC Berkeley
	Combinatorics of m=1 Grasstopes
	A Grasstope is a linear projection of the totally nonnegative Grassmannian to a smaller Grassmannian. This is a generalization of the amplituhedron, a geometric object of great importance to calculating scattering amplitudes in physics. The amplituhedron is a Grasstope arising from a totally positive linear map. While amplituhedra are relatively well-studied, much less is known about general Grasstopes. In this talk, I will discuss combinatorics and geometry of Grasstopes in the m=1 case. In particular, I will show that they can be characterized as unions of cells of a hyperplane arrangement satisfying a certain sign variation condition and argue that amplituhedra are (in a certain sense) minimal Grasstopes. This is based on joint work with Dmitrii Pavlov and Lizzie Pratt.
5:30 – 6:30 pm	Welcome Reception

Thursday, May 30, 2024

8:30 – 9:00 am	Breakfast
9:00 – 10:00 am	Nima Arkani-Hamed, IAS
	Title/Abstract TBA
10:00 – 10:30 am	Coffee Break
10:30 – 11:20 am	Hugh Thomas, Université du Québec à Montréal
	u-equations from finite dimensional algebras
	In this talk, I will explain how to write down and solve a system of u-equations associated to any finite dimensional algebra with finitely many indecomposable representations. These vastly generalize the system of equations written down by Koba and Nielsen in 1969, which from our point of view are associated to the representation theory of a Dynkin type A quiver. I will discuss features of the resulting solution spaces, including connections to tau-tilting theory, and the relationships that exist among different spaces of solutions. I will also say something about how different choices of finite-dimensional algebra put us in (i) the setting of cluster algebras, (ii) the Grassmannian combinatorics of non-kissing complexes, or (iii) the curves-on-surfaces model directly relevant to amplitudes. This talk reports on joint work with Nima Arkani-Hamed, Hadleigh Frost, Pierre-Guy Plamondon, and Giulio Salvatori.
11:20 – 11:30 am	Coffee Break
11:30 – 12:20 pm	Dani Kaufman, University of Copenhagen
	Affine Cluster Algebras
	Affine cluster algebras form the simplest examples of non-finite type cluster algebras. While they have infinitely many clusters, they are still mutation finite and have essentially one mutation sequence which produces infinitely many clusters. I will give an introduction to these cluster algebras by comparing them with finite cluster algebras. I will also show how some structures similar to finite type cluster algebras appear "in the limit" along this infinite mutation sequence. If time I will also mention how the "infinite cluster variables" which live in the limit are related to the algebraic letters appearing in the symbol alphabet for 8 particle N=4 SYM amplitudes.
12:30 – 12:45	Group Photo: 20 Garden Street, front entrance stairs outside building
12:45 – 2:00 pm	Lunch Break
2:00 – 2:50 pm	Ran Tessler, Weizmann Institute of Science
	The magic number for the m=2 amplituhedron
	We will start by reviewing the amplituhedron and its tilings. We will then show that all tilings of the m=2 amplituhedron have the same cardinality ("the magic number"), proving the m=2 case of a conjecture that the same holds for all even-m amplituhedra. If time permits we will discuss related results and consequences. Based on a joint work with Parisi, Sherman-Bennett and Williams.

| 2:50 – 3:00 pm | Coffee Break |
| 3:00 – 3:50 pm | Melissa Sherman-Bennett, MIT & UC Davis |

Cluster algebras and tilings of amplituhedra

Physicists Arkani-Hamed and Trnka introduced the amplituhedron to better understand scattering amplitudes in N=4 super Yang-Mills theory. The amplituhedron is the image of the totally nonnegative Grassmannian under the "amplituhedron map". Examples of amplituhedra include cyclic polytopes, the totally nonnegative Grassmannian itself, and cyclic hyperplane arrangements. Of primary interest to physics are tilings of amplituhedra, which are roughly analogous to subdivisions of polytopes. I will discuss joint work with Even-Zohar, Lakrec, Parisi, Tessler and Williams on BCFW tilings of m=4 amplituhedra and the surprising connection between these tilings and the cluster algebra structure of the Grassmannian.

3:50 – 4:00 pm	Coffee Break
4:00 – 5:30 pm	Open Problems Forum
6:00 – 8:00 pm	Conference Dinner

Friday, May 31, 2024

| 8:30 – 9:00 am | Breakfast |
| 9:00 – 10:00 am | Yu-tin Huang, National Taiwan University |

Chambers and all loop geometry for four-point correlators

The all loop amplituhedron for N=4 SYM (and ABJM theory) can be recast into the notion of loop fibration over tree geometry. This leads to a further dissection of the tree geometry into "chambers", whose boundaries represents when the associated loop-form changes. In this talk I will present a new geometry associated with the all loop four-point correlator of N=4 SYM, where similar description is present. Interestingly, at four-loops, this gives a first example where the chamber form is rational even though it's loop form contains elliptic integrals.

10:00 – 10:15 am	Coffee Break
10:15 – 12:30 am	Emerging Scholar Talks
10:15 – 10:40 am	Evgeniya Akhmedova, Weizmann Institute of Science (via Zoom)

The tropical amplituhedron

The Amplituhedron is a geometric object discovered recently by Arkani-Hamed and Trnka, that provides a completely new direction for calculating scattering amplitudes in quantum field theory. We define a tropical analogue of this object, the tropical amplituhedron and study its structure and boundaries. It can be considered as both the tropical limit of the amplituhedron and a generalization of the tropical positive Grassmannian.

| 10:40 – 11:10 am | Elizabeth Pratt, UC Berkeley |

The Chow-Lam Form

The classical Chow form encodes any projective variety by one equation. We introduce the Chow-Lam form for subvarieties of a Grassmannian. By evaluating the Chow-Lam form at twistor coordinates, we obtain universal projection formulas, which were pioneered by Thomas Lam for positroid varieties in the study of amplituhedra. This is joint work with Bernd Sturmfels.

| 11:10 – 11:30 am | Sebastian Seemann, KU Leuven |

Vandermonde cells as positive geometries

Vandermonde cells represent semialgebraic subsets of R^n, characterized as the image of a simplex under the Vandermonde map. However, within the realm of positive geometry, several challenges arise in establishing canonical forms for these cells. These include issues such as non-normal boundaries, non-transversal intersections, and singularities of boundary curves. Even more difficulties appear when considing the limiting Vandermonde cell, which is not semi-algebraic and thus doesn't fit within the standard framework of positive geometries. In this presentation, I will first review the notion of Polypols and their canonical forms, examining the complexities encountered when dealing with Vandermonde cells. In particular, I will explain what goes wrong in the case of Vandermonde cells and which obstructions we can deal with.

11:30 – 11:40 am	Coffee break
11:40 – 12:10 pm	Chia-Kai Kuo, National Taiwan University *Geometric transition from maximal SYM to ABJM* Recently, the ABJM amplituhedron has been proposed, encoding all-loop and all-multiplicity ABJM amplitudes. It is constructed by slightly modifying the original definition. In this talk, I will explore the significance of these modifications in transitioning theoretical models from super Yang-Mills theory to ABJM theory. A key focus will be on how symplectic reduction and the overall sign change in the positivity conditions ensure the consistency of ABJM amplitudes. Additionally, I will discuss some distinct features of this geometry.
12:10 – 12:30 pm	Lecheng Ren, Brown University *Symbol alphabets from tensor diagrams* We propose to use tensor diagrams and the Fomin-Pylyavskyy conjectures to explore the connection between symbol alphabets of n-particle amplitudes in planar N= 4 Yang-Mills theory and certain polytopes associated to the Grassmannian Gr(4, n). We show how to assign a web (a planar tensor diagram) to each facet of these polytopes. Webs with no inner loops are associated to cluster variables (rational symbol letters). For webs with a single inner loop we propose and explicitly evaluate an associated web series that contains information about algebraic symbol letters. In this manner we reproduce the results of previous analyses of $n \leq 8$, and find that the polytope C(4,9) encodes all rational letters, and all square roots of the algebraic letters, of known nine-particle amplitudes.
12:30 – 2:00 pm	Lunch Break
2:00 – 2:50 pm	Paolo Benincasa, MPI (via Zoom) *Cosmological Polytopes & Beyond* Together with being the source of the most profound questions in fundamental physics, cosmology turns out to be an arena from where novel combinatorial structures emerge. In this talk, I will give a gentle introduction to the cosmological polytopes, describing the so-called Bunch-Davies wavefunction for a large class of scalar theories, and how it can be used to define and characterize less conventional objects, named optical polytopes and weighted cosmological polytopes, which provide examples of non-convex and weighted geometries respectively.
2:50 – 3:00 pm	Coffee Break
3:00 – 3:45 pm	Shruti Paranjape, UC Davis *Loops in a loop expansion* In a paper by Arkani-Hamed, Henn and Trnka, it was shown that the amplituhedron construction of N=4 SYM can be recast in terms of negative geometries with a certain hierarchy of loops (closed cycles) in the space of loop momentum twistors. Furthermore, using differential equation methods, it was possible to calculate and resum integrated expressions and obtain strong coupling results. In this talk, we provide a more general framework for the loops of loops expansion and outline a powerful method for the determination of differential forms for higher-order geometries. In particular, we will focus on the case of 1 closed cycle in loop space and select integrated results.
3:45 – 4:30 pm	Nick Early, Weizmann Institute of Science *Minimal Kinematics on $\mathcal{M}_{0,n}$, and beyond* Minimal Kinematics (MK) identifies kinematic degenerations of the CHY scattering potential where the critical points are given by rational formulas. These rest on the Horn uniformization of Kapranov-Huh; they are specified combinatorially by 2-trees. On the other hand, Planar Kinematics (PK) identifies the locus in $M_{0,n}$ which is fixed by cyclic permutation. Combining MK and PK realizes a maximally thin relative of the associahedron known as the PK polytope; it is a reflexive polytope, and its polar dual, the root polytope, has volume a Catalan number. In this talk, we start by exploring MK and PK on the moduli space $M_{0,n}$. We explain how this story generalizes to moduli spaces $X(k,n)$ of points in projective space $\mathbb{P}^{k-1}$, to CEGM amplitudes and beyond.
4:30 – 5:00 pm	Coffee and Farewell