Deformation of Geometric Structures in Current Mathematics: A Celebration of the Works of Masatake Kuranishi

May 9–12, 2022

Monday, May 9, 2022

8:15 am	Light breakfast & coffee/tea	
8:45–9:00 am	Opening Remarks	
9:00–10:00 am	Kenji Fukaya	**Title:** Gromov Hausdorff convergence of filtered A infinity category **Abstract:** In mirror symmetry a mirror to a symplectic manifold is actually believed to be a family of complex manifold parametrized by a disk (of radius 0). The coordinate ring of the parameter space is a kind of formal power series ring the Novikov ring. Novikov ring is a coefficient ring of Floer homology. Most of the works on homological Mirror symmetry so far studies A infinity category over Novikov field, which corresponds to the study of generic fiber. The study of A infinity category over Novikov ring is related to several interesting phenomenon of Hamiltonian dynamics. In this talk I will explain a notion which I believe is useful to study mirror symmetry.
10:15–11:15 am	Nigel Hitchin (Zoom)	**Title:** Deformations: a personal perspective **Abstract:** The talk, largely historical, will focus on different deformation complexes I have encountered in my work, starting with instantons on 4-manifolds, but also monopoles, Higgs bundles and generalized complex structures. I will also discuss some speculative ideas related to surfaces of negative curvature.
11:30–12:30 pm	H. Blaine Lawson	**Title:** Projective Hulls, Projective Linking, and Boundaries of Varieties **Abstract:** In 1958 John Wermer proved that the polynomial hull of a compact real analytic curve $\g\ss\bbc^n$ was a 1-dim'l complex subvariety of $\bbc^n-\g$. This % fundamental result engendered much subsequent activity, and was related to Gelfand's spectrum of a Banach algebra. In the early 2000's Reese Harvey and I found a projective analogue of these concepts and wondered whether Wermer's Theorem could be generalized to the projective setting. This question turned out to be more subtle and quite intriguing, with unexpected consequences. We now know a

		great deal, a highpoint of which s a result with Harvey and Wermer. It led to conjectures (for C^ω-curves in $\bbp^2_\bbc$) which imply several results. One says, roughly, that a $(2p-1)$-cycle $\G$ in $\bbp^n$ bounds a positive holomorphic p-chain of mass $\leq \L$ \ $\iff$\ its normalized linking number with all positive $(n-p)$-cycles in $\bbp^n-\|\G\|$ is $\geq -\L$. Another says that a class $\tau\in H_{2p}(\bbp^n, \|\G\|; \bbz)$ with $\partial \tau=\G$ contains a positive holomorphic p-chain \ $\iff$ \ $\tau\bullet [Z]\geq 0$ for all positive holomorphic $(n-p)$-cycles Z in $\bbp^n – \|\G\|$.
12:30–2:30 pm	Lunch Break	
2:30–3:30 pm	Gabor Szekelyhidi	**Title:** Singularities along the Lagrangian mean curvature flow. **Abstract:** We study singularity formation along the Lagrangian mean curvature flow of surfaces. On the one hand we show that if a tangent flow at a singularity is the special Lagrangian union of two transverse planes, then the flow undergoes a “neck pinch”, and can be continued past the flow. This can be related to the Thomas-Yau conjecture on stability conditions along the Lagrangian mean curvature flow. In a different direction we show that ancient solutions of the flow, whose blow-down is given by two planes meeting along a line, must be translators. These are joint works with Jason Lotay and Felix Schulze.
3:30–4:00 pm	Coffee Break	
4:00–5:00 pm	Takeo Ohsawa	**Title:** Glimpses of embeddings and deformations of CR manifolds **Abstract:** Basic results on the embeddings and the deformations of CR manifolds will be reviewed with emphasis on the reminiscences of impressive moments with Kuranishi since his visit to Kyoto in 1975.

Tuesday, May 10, 2022

8:15 am	Light breakfast & coffee/tea	
9:00–10:00 am	Charles Fefferman (Zoom)	**Title:** Interpolation of Data by Smooth Functions **Abstract:** Let X be your favorite Banach space of continuous functions on R^n. Given an (arbitrary) set E in R^n and an arbitrary function f:E->R, we ask: How can we tell whether f extends to a function F \in X? If such an F exists, then how small can we take its norm? What can we say about its derivatives (assuming functions in X have derivatives)? Can we take F to depend linearly on f? Suppose E is finite. Can we compute an F as above with norm nearly as small as possible? How many computer operations does it take? What if F is required to agree only approximately with f on E? What if we are allowed to discard a few data points (x, f(x)) as "outliers"? Which points should we discard? The results were obtained jointly with A. Israel, B. Klartag, G.K. Luli and P. Shvartsman over many years.
10:15–11:15 am	Claire Voisin	**Title:** Deformations of K-trivial manifolds and applications to hyper-Kähler geometry **Summary:** I will explain the Ran approach via the T^1-lifting principle to the BTT theorem stating that deformations of K-trivial compact Kähler manifolds are unobstructed. I will explain a similar unobstructedness result for Lagrangian submanifolds of hyper-Kähler manifolds and I will describe important consequences on the topology and geometry of hyper-Kähler manifolds.
11:30– 2:30 pm	Victor Guillemin	**Title:** Semi-Classical Functions of Isotropic Type **Abstract:** The world of semiclassical analysis is populated by objects of "Lagrangian type." The topic of this talk however will be objects in semi-classical analysis that live instead on isotropic submanifolds. I will describe in my talk a lot of interesting examples of such objects.
12:30–2:30 pm	Lunch Break	
2:30–3:30 pm	Teng Fei	**Title:** Symplectic deformations and the Type IIA flow **Abstract:** The equations of flux compactification of Type IIA superstrings were written down by Tomasiello and Tseng-Yau. To study these equations, we introduce a natural geometric flow

		known as the Type IIA flow on symplectic Calabi-Yau 6-manifolds. We prove the wellposedness of this flow and establish the basic estimates. We show that the Type IIA flow can be applied to find optimal almost complex structures on certain symplectic manifolds. We prove the dynamical stability of the Type IIA flow, which leads to a proof of stability of Kahler property for Calabi-Yau 3-folds under symplectic deformations. This is based on joint work with Phong, Picard and Zhang.
Speakers Banquet		

Wednesday, May 11, 2022

8:15 am	Light breakfast & coffee/tea	
9:00–10:00 am	Shing-Tung Yau (Zoom)	**Title:** Canonical metrics and stability in mirror symmetry **Abstract:** I will discuss the deformed Hermitian-Yang-Mills equation, its role in mirror symmetry and its connections to notions of stability. I will review what is known, and pose some questions for the future.
10:15–11:15 am	Duong H. Phong	**Title:** $L^\\infty$ estimates for the Monge-Ampere and other fully non-linear equations in complex geometry **Abstract:** A priori estimates are essential for the understanding of partial differential equations, and of these, $L^\\infty$ estimates are particularly important as they are also needed for other estimates. The key $L^\\infty$ estimates were obtained by S.T. Yau in 1976 for the Monge-Ampere equation for the Calabi conjecture, and sharp estimates obtained later in 1998 by Kolodziej using pluripotential theory. It had been a long-standing question whether a PDE proof of these estimates was possible. We provide a positive answer to this question, and derive as a consequence sharp estimates for general classes of fully non-linear equations. This is joint work with B. Guo and F. Tong.
11:30–2:30 pm	Paul Seidel	**Title:** The quantum connection: familiar yet puzzling **Abstract:** The small quantum connection on a Fano variety is possibly the most basic piece of enumerative geometry. In spite of being really easy to write down, it is the subject of far-reaching conjectures (Dubrovin, Galkin, Iritani), which challenge our understanding of mirror symmetry. I will give a gentle introduction to the simplest of these questions.
12:30–2:30 pm	Lunch Break	
2:30–3:30 pm	Melissa C.C. Liu	**Title:** Higgs-Coulumb correspondence for abelian gauged linear sigma models **Abstract:** The underlying geometry of a gauged linear sigma model (GLSM) consists of a GIT quotient of a complex vector space by the linear action of a reductive algebraic group G (the gauge group) and a polynomial function (the superpotential) on the GIT quotient. The Higgs-Coulomb correspondence relates (1) GLSM invariants which are virtual counts of curves in the critical locus of the superpotential (Higgs branch), and (2) Mellin-Barnes type integrals on the Lie algebra of G (Coulomb branch). In this

		talk, I will describe the correspondence when G is an algebraic torus, and explain how to use the correspondence to study dependence of GLSM invariants on the stability condition. This is based on joint work with Konstantin Aleshkin.
3:30–4:00 pm	Coffee Break	
4:00–5:00 pm	Sebastien Picard	**Title:** Topological Transitions of Calabi-Yau Threefolds **Abstract:** Conifold transitions were proposed in the works of Clemens, Reid and Friedman as a way to travel in the parameter space of Calabi-Yau threefolds with different Hodge numbers. This process may deform a Kahler Calabi-Yau threefold into a non-Kahler complex manifold with trivial canonical bundle. We will discuss the propagation of differential geometric structures such as balanced hermitian metrics, Yang-Mills connections, and special submanifolds through conifold transitions. This is joint work with T. Collins, S. Gukov and S.-T. Yau.

Thursday, May 12, 2022

8:15 am	Light breakfast & coffee/tea	
9:00 am–10:00 am	Akito Futaki (Zoom)	**Title:** Transverse coupled K\"ahler-Einstein metrics and volume minimization **Abstract:** We show that transverse coupled K\"ahler-Einstein metrics on toric Sasaki manifolds arise as a critical point of a volume functional. As a preparation for the proof, we re-visit the transverse moment polytopes and contact moment polytopes under the change of Reeb vector fields. Then we apply it to a coupled version of the volume minimization by Martelli-Sparks-Yau. This is done assuming the Calabi-Yau condition of the K\"ahler cone, and the non-coupled case leads to a known existence result of a transverse K\"ahler-Einstein metric and a Sasaki-Einstein metric, but the coupled case requires an assumption related to Minkowski sum to obtain transverse coupled K\"ahler-Einstein metrics.
10:15 am–11:15 am	Yu-Shen Lin	**Title:** SYZ Mirror Symmetry of Log Calabi-Yau Surfaces **Abstract:** Strominger-Yau-Zaslow conjecture predicts Calabi-Yau manifolds admits special Lagrangian fibrations. The conjecture serves as one of the guiding principles in mirror symmetry. In this talk, I will explain the existence of the special Lagrangian fibrations in some log Calabi-Yau surfaces and their dual fibrations in their expected mirrors. The journey leads us to the study of the moduli space of Ricci-flat metrics with certain asymptotics on these geometries and the discovery of new semi-flat metrics. If time permits, I will explain the application to the Torelli theorem of ALH^* gravitational instantons. The talk is based on joint works with T. Collins and A. Jacob.
11:30 am – 12:30 pm	Robert Friedman	**Title:** Deformations of singular Fano and Calabi-Yau varieties **Abstract:** This talk will describe recent joint work with Radu Laza on deformations of generalized Fano and Calabi-Yau varieties, i.e. compact analytic spaces whose dualizing sheaves are either duals of ample line bundles or are trivial. Under the assumption of isolated hypersurface canonical singularities, we extend results of Namikawa and Steenbrink in dimension three and discuss various generalizations to higher dimensions.
12:30 pm	Concluding Remarks	