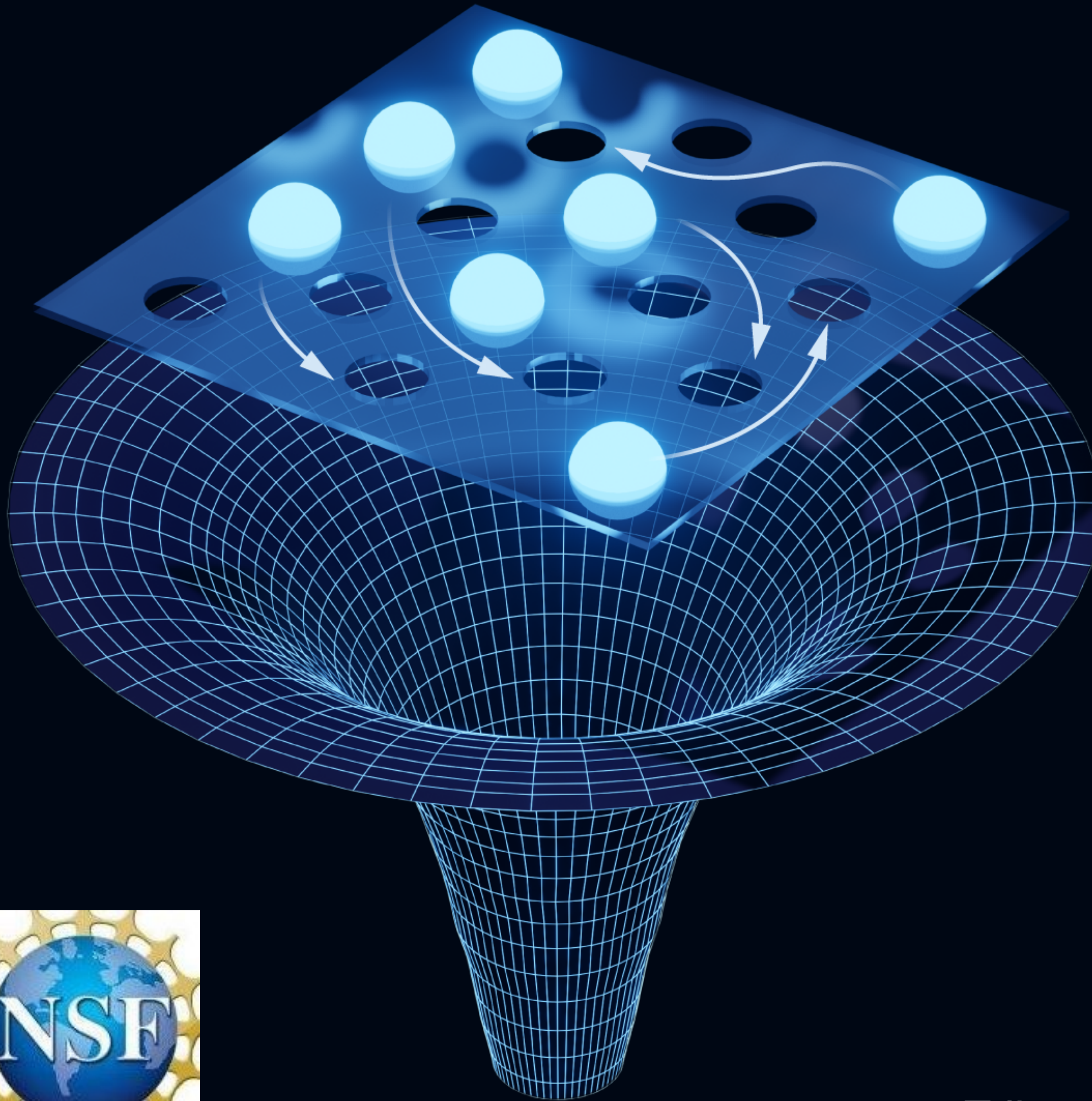


Quantum statistical mechanics of charged black holes and strange metals



CMSA, Harvard
October 5, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



SYK
model

The Sachdev-Ye-Kitaev (SYK) model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

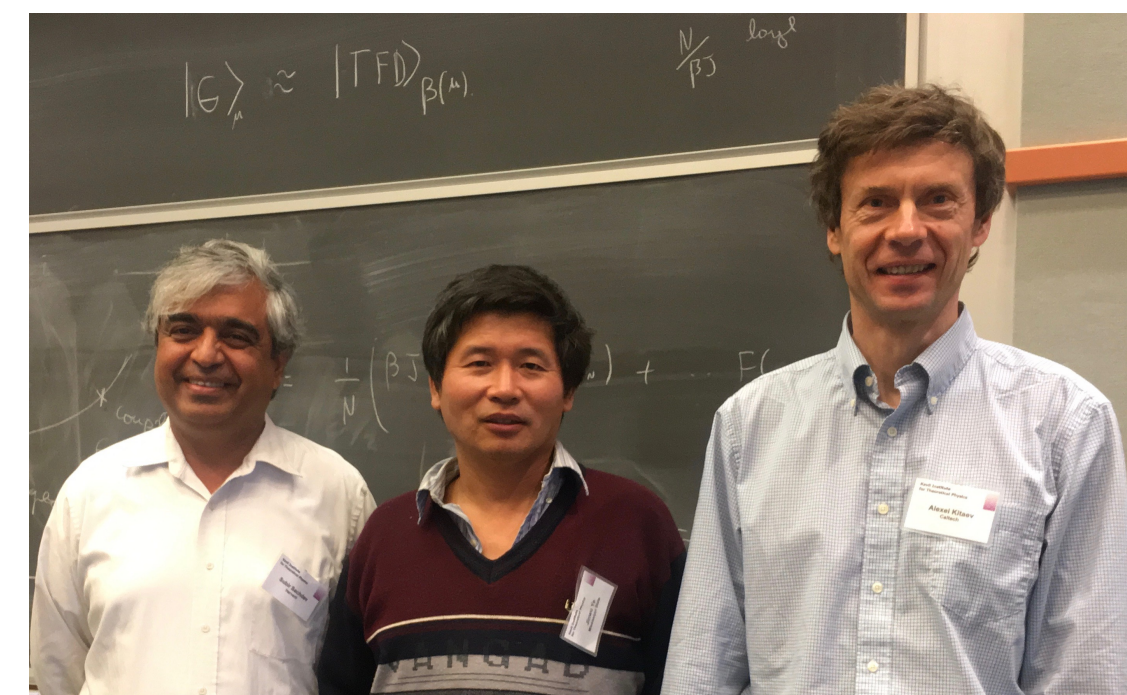
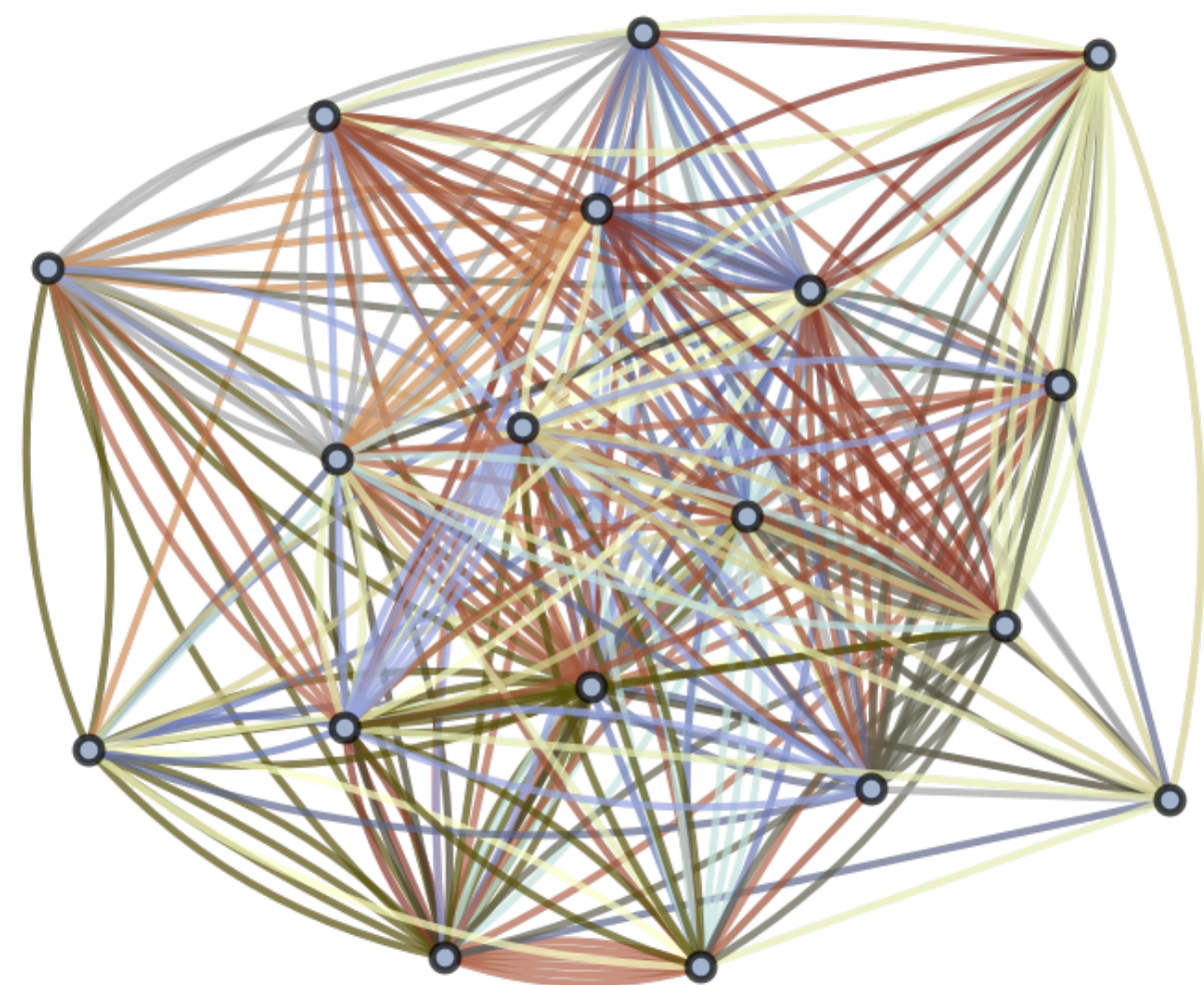
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



The Sachdev-Ye-Kitaev (SYK) model

$$\mathcal{Z}(Q, T) = \text{Tr}_Q \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T); \text{ Entropy } S = -\frac{\partial F}{\partial T}.$$

The Sachdev-Ye-Kitaev (SYK) model

$$\begin{aligned} \mathcal{Z}(Q, T) &= \text{Tr}_{\mathcal{Q}} \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T); \text{ Entropy } S = -\frac{\partial F}{\partial T}. \\ &\equiv \int_{E_0}^{\infty} D(E) e^{-E/T}; \quad D(E) = \sum_i \delta(E - E_i); \quad \mathcal{H} |\Psi_i\rangle = E_i |\Psi_i\rangle \end{aligned}$$

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Main result from effective theory of quantum gravity

$$D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E}); \quad S = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{N^{1/3} U}{T}\right)$$

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

J. S. Cotler et al., JHEP 05 (2017) 118

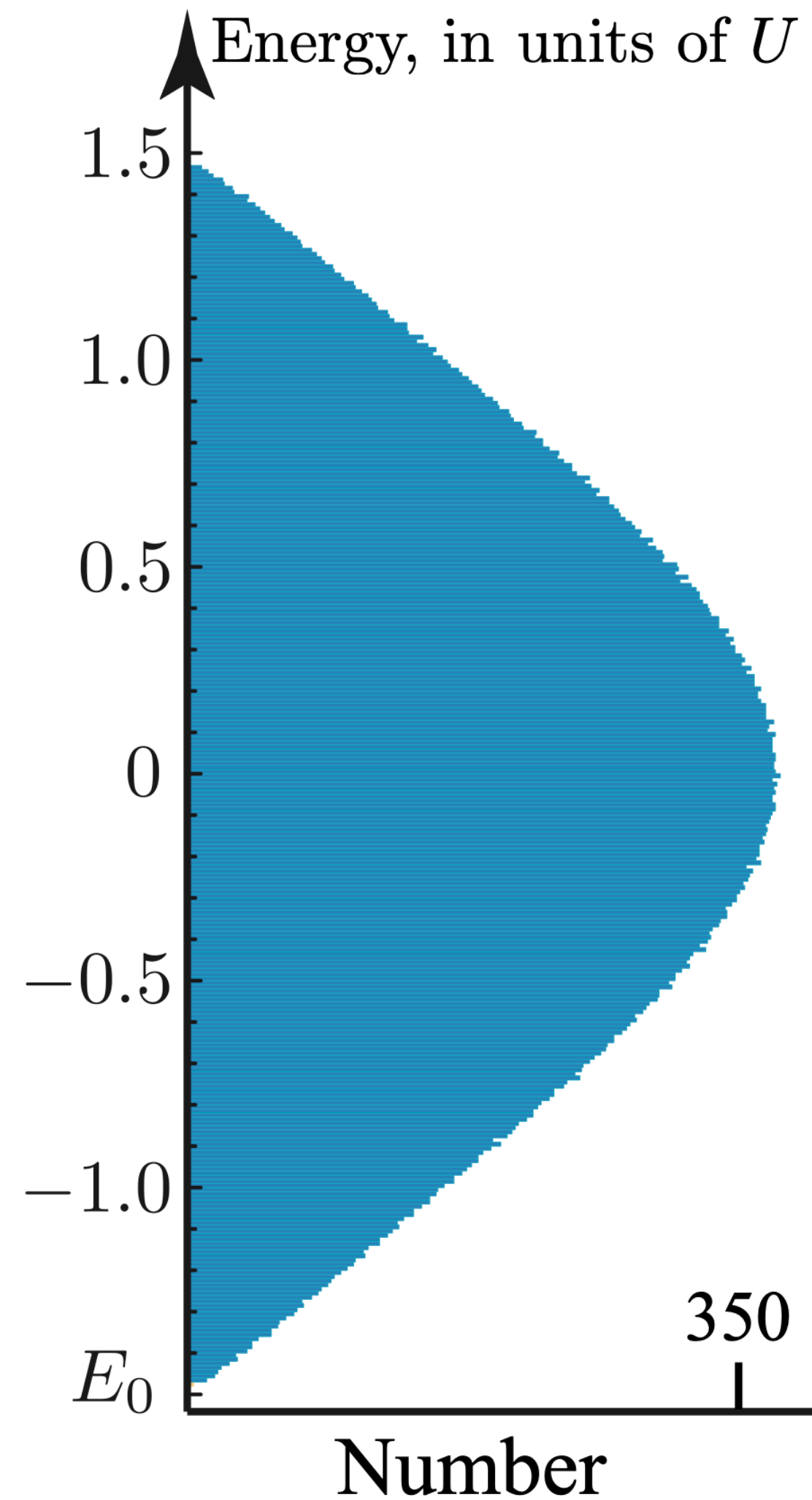
At $\mathcal{Q} = 1/2$, we have

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833 \dots$$

and $\gamma = \# / U$ is non-universal

Many-body density of states

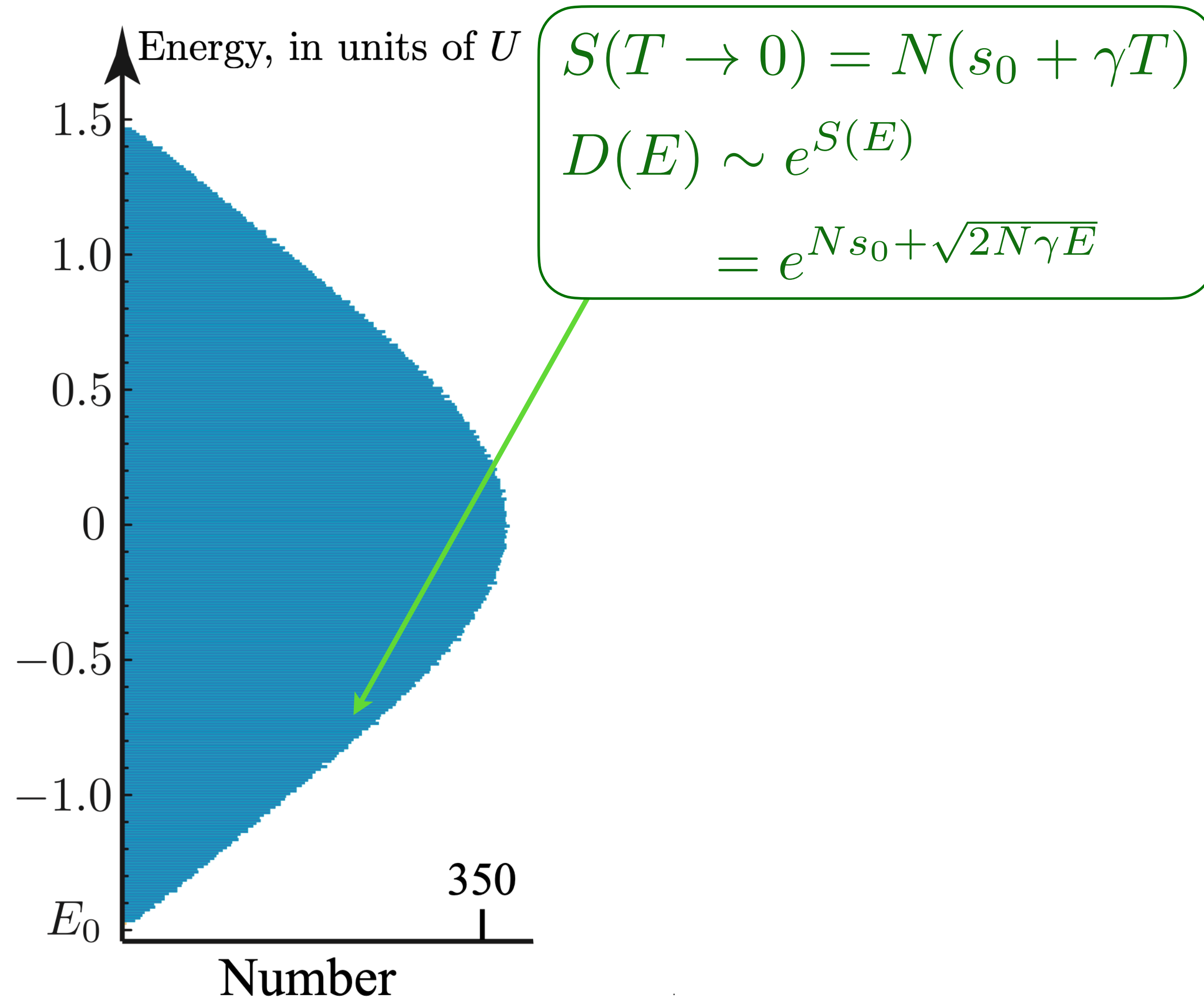
$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

Many-body density of states

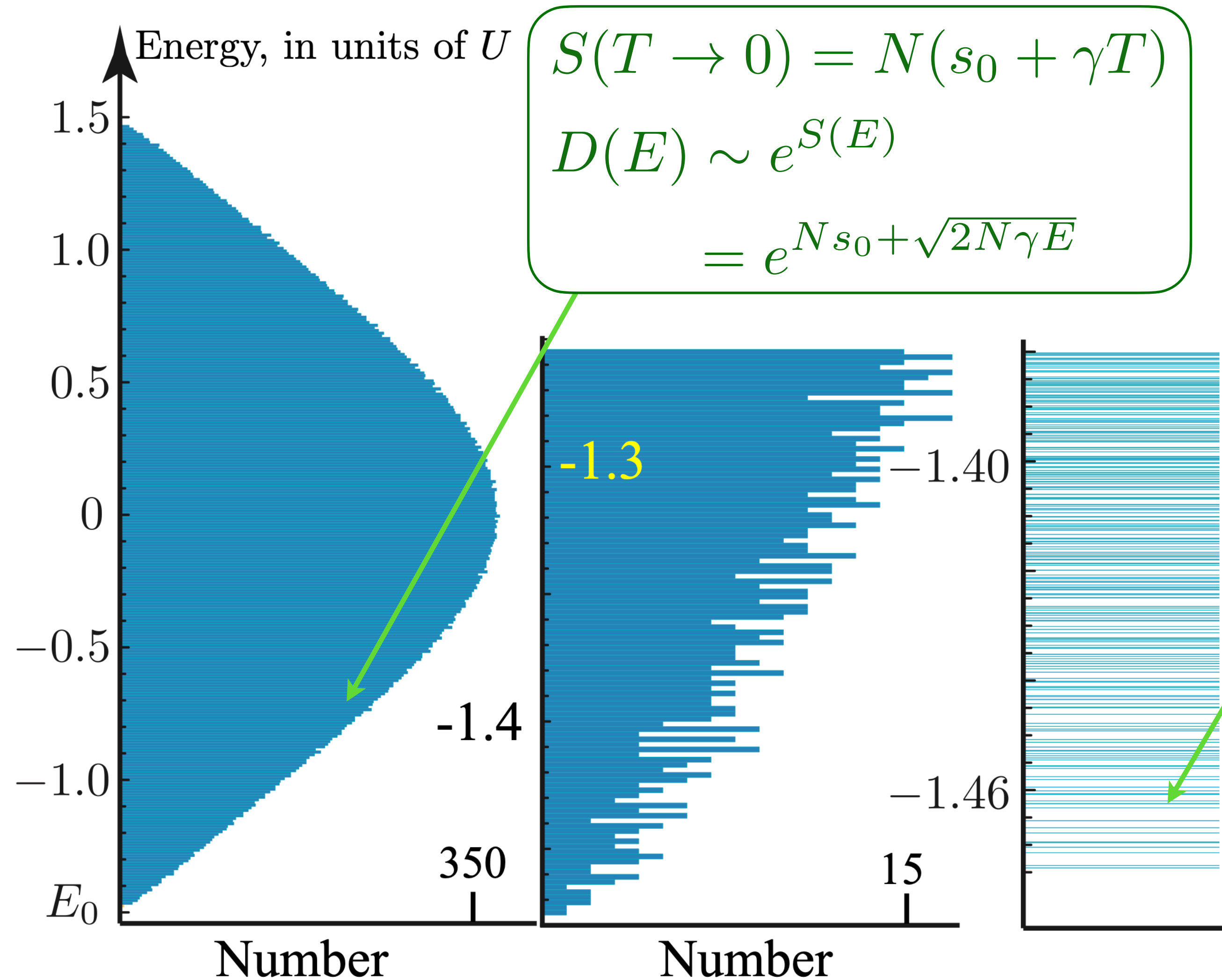
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Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

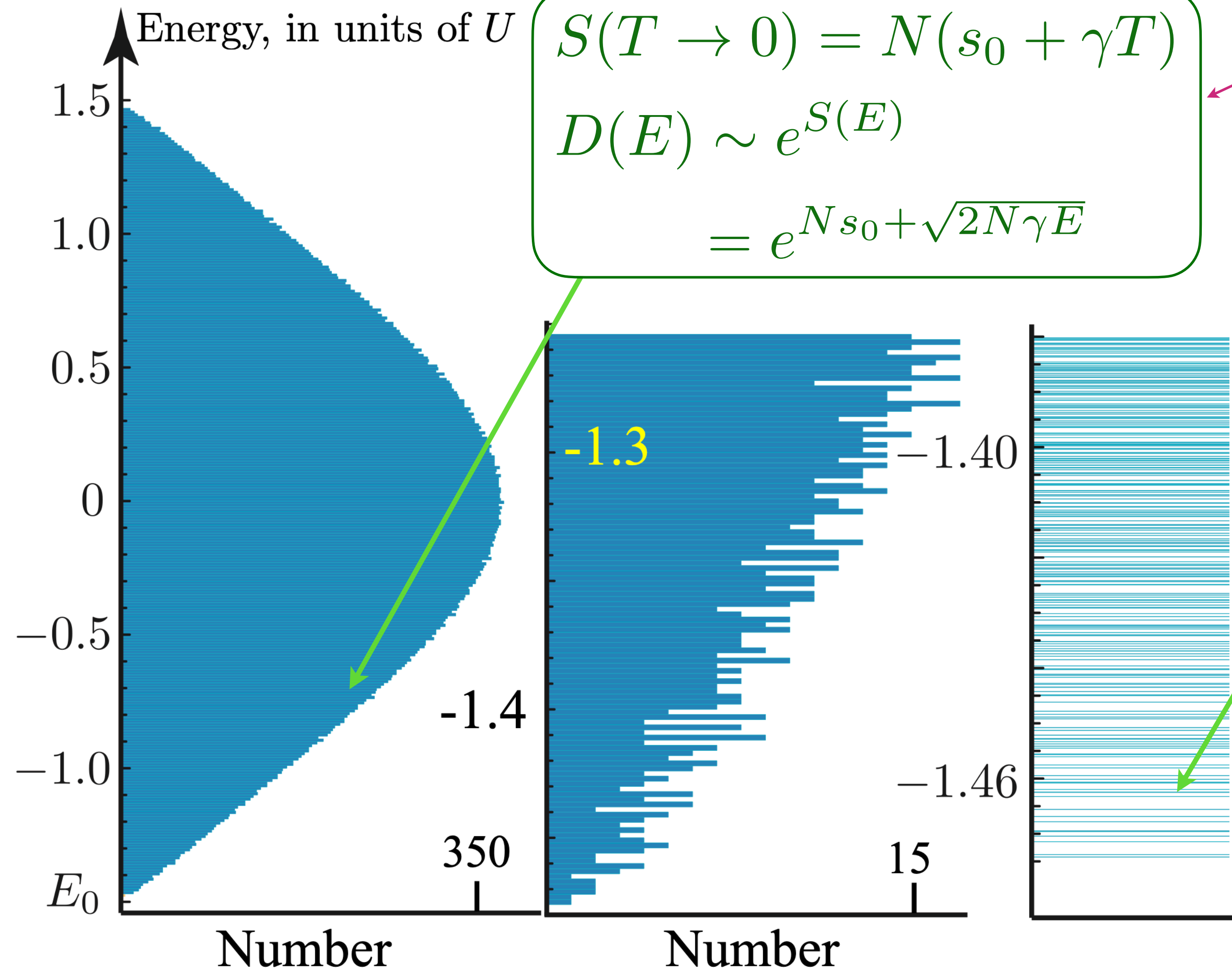
$$D(E) \sim e^{N s_0} \sqrt{2N\gamma E}$$

No exponentially large degeneracy, but exponentially small level spacing!
 No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

G-Σ path integral

After introducing replicas $a = 1 \dots n$, and integrating out the disorder, the partition function can be written as

$$\mathcal{Z} = \int \mathcal{D}c_{ia}(\tau) \exp \left[- \sum_{ia} \int_0^\beta d\tau c_{ia}^\dagger \left(\frac{\partial}{\partial \tau} - \mu \right) c_{ia} - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{ia}^\dagger(\tau) c_{ib}(\tau') \right|^4 \right].$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \left[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left(G(\tau_2, \tau_1) + \frac{1}{N} \sum_i c_i(\tau_2) c_i^\dagger(\tau_1) \right) \right].$$

G - Σ
path
integral

Then the partition function can be written as a path integral with an action I analogous to a Luttinger-Ward functional

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NI)$$
$$I = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)]$$
$$+ \int d\tau_1 d\tau_2 [\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5**, 041025 (2015)

G - Σ
path
integral

Reparametrization and phase zero modes

We find the saddle point, G_s , Σ_s , and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-E_0/T + N s_0 - N I_{\text{eff}}[f, \phi]},$$

where $E_0 \propto N$ is the ground state energy.

Low temperature thermodynamics: for $k_B T \ll U$

$$\mathcal{Z} = \text{Tr} \exp \left(-\frac{\mathcal{H}}{k_B T} \right) \\ \approx \exp \left(N \frac{s_0}{k_B} \right) \int \frac{\mathcal{D}f(\tau) \mathcal{D}\phi(\tau)}{||\text{SL}(2, \mathbb{R})||} \exp \left(-\frac{1}{\hbar} I_{\text{eff}} [f(\tau), \phi(\tau)] \right)$$

$$I_{\text{eff}} [f, \phi] = \frac{NK}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau f)^2 - \frac{N\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K , γ , and \mathcal{E} can be related to thermodynamic derivatives, and we have used the Schwarzian:

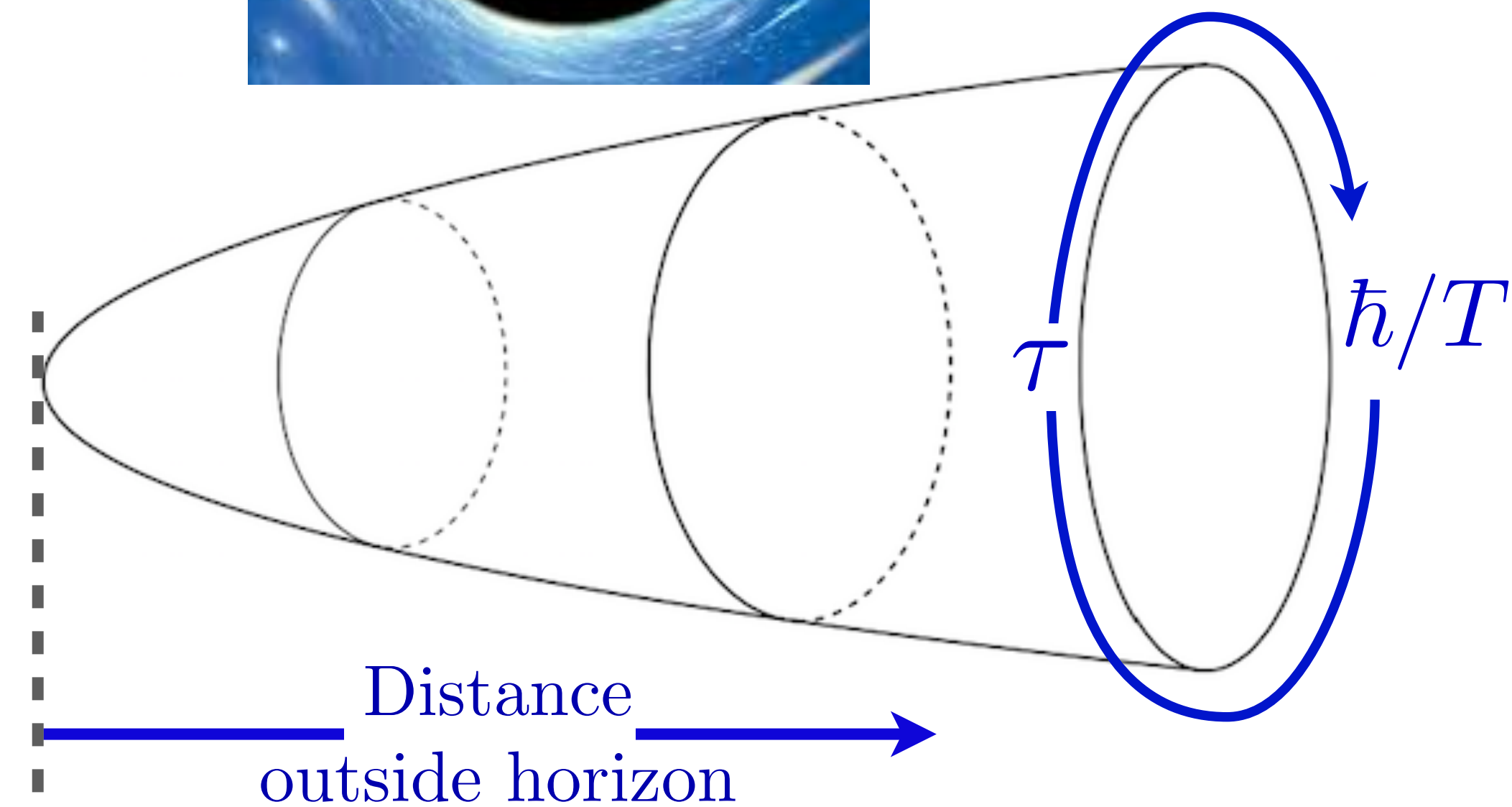
$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left(\frac{g''}{g'} \right)^2.$$

**Charged
black holes**

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$



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$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

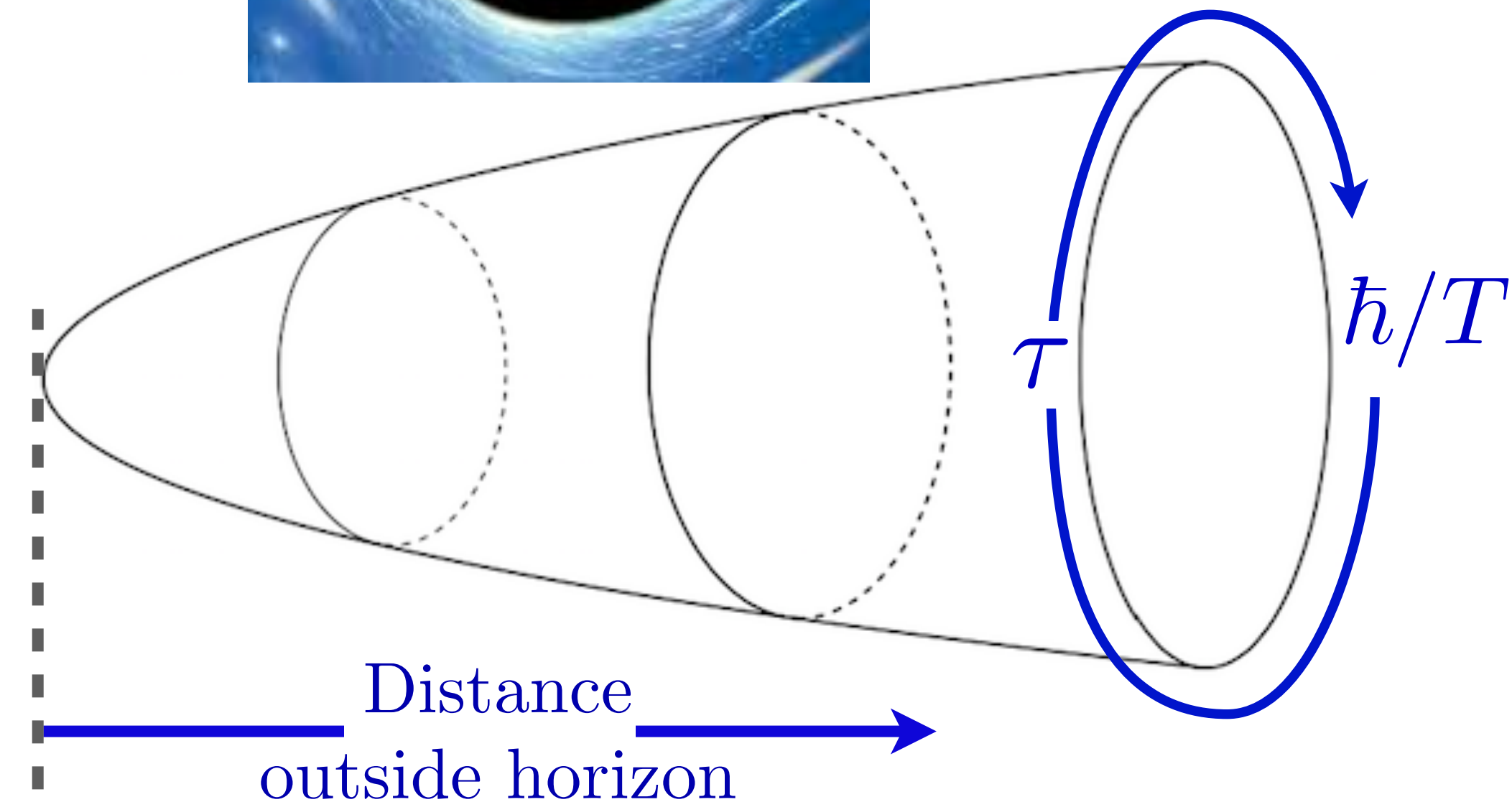
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



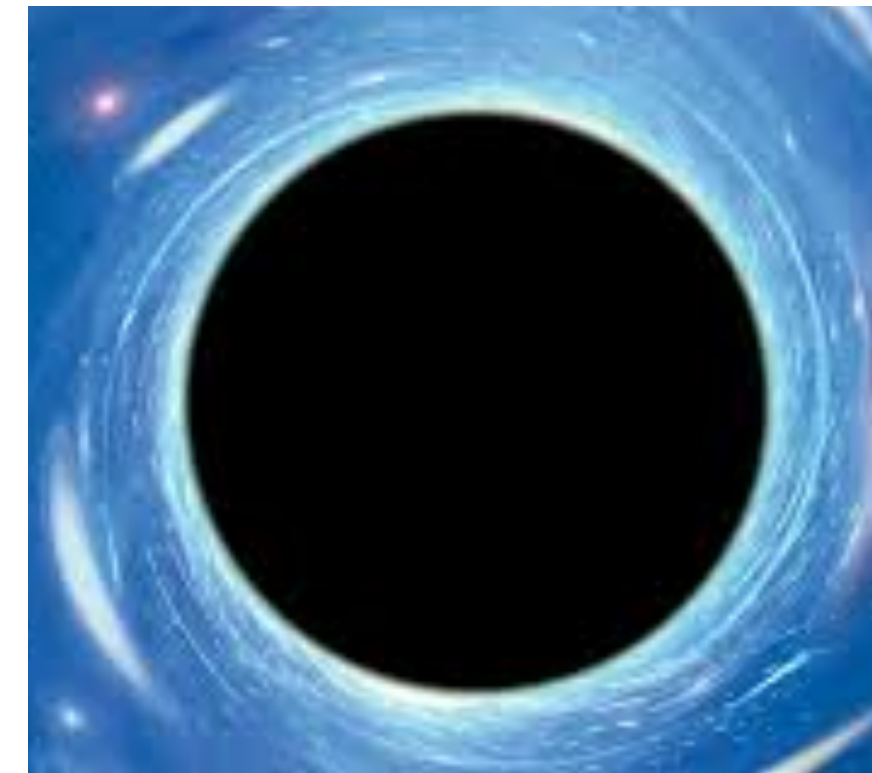
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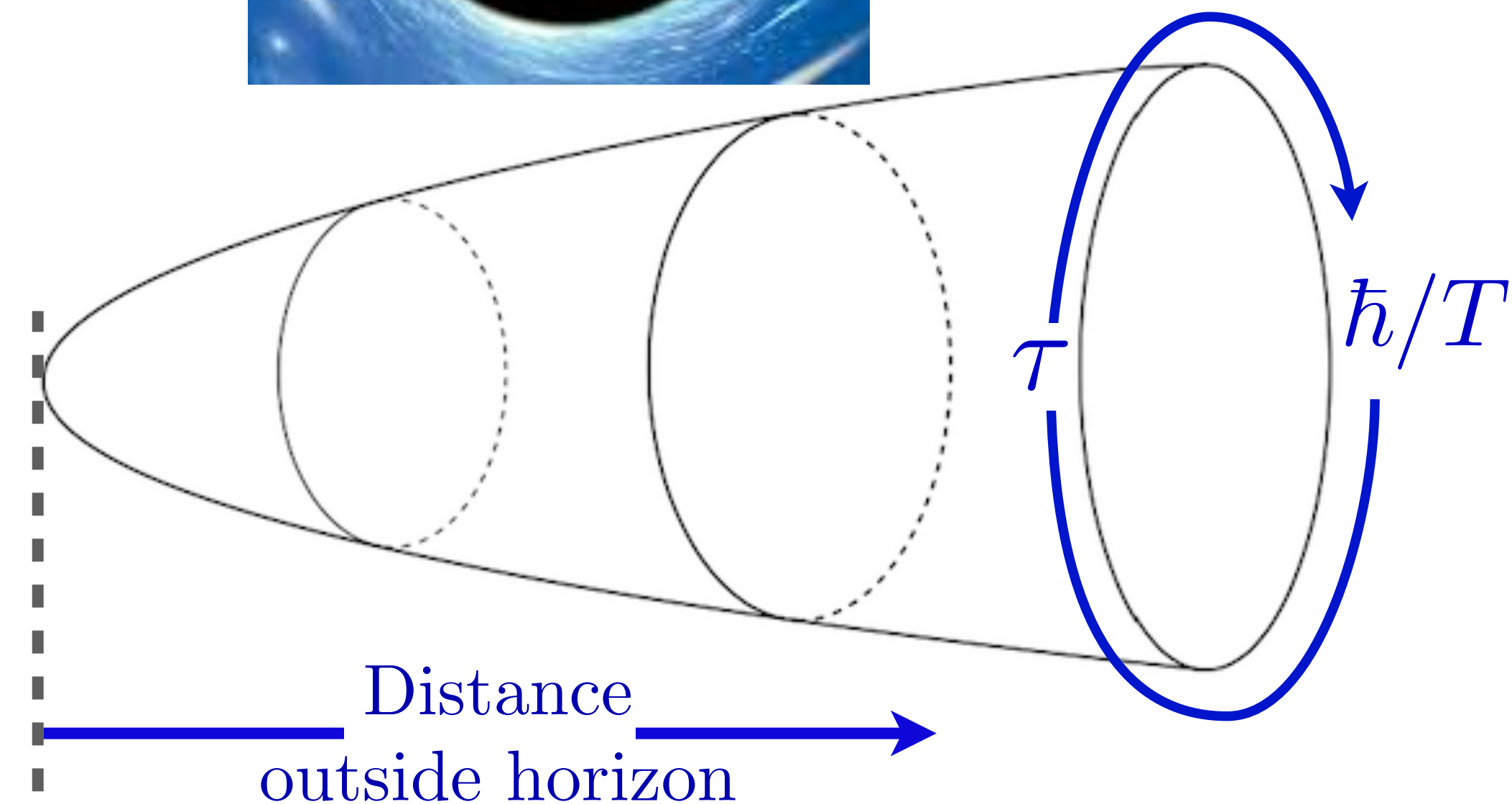


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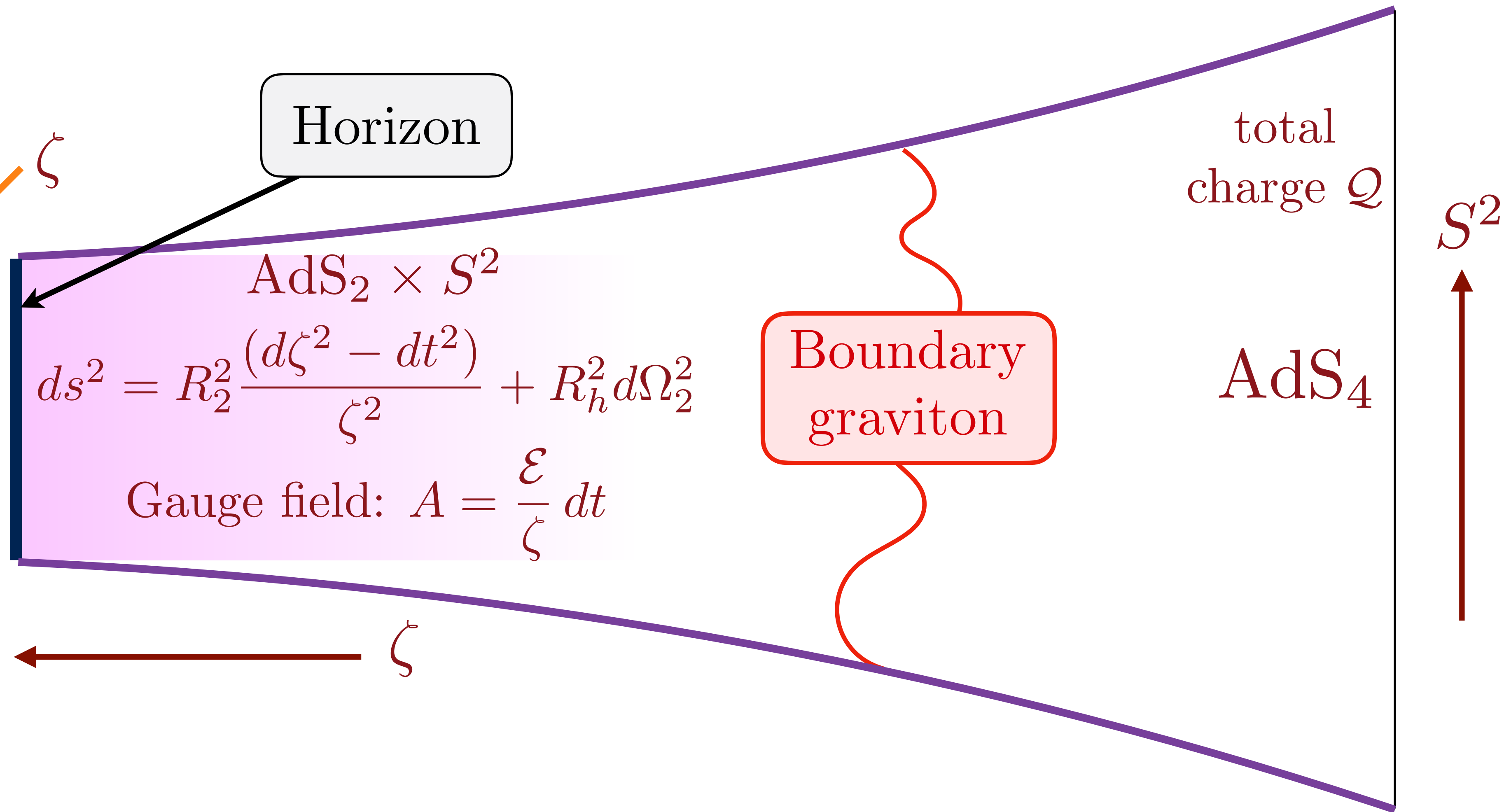
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Note the similarity to the large N entropy of the SYK model!
(along with other similarities)

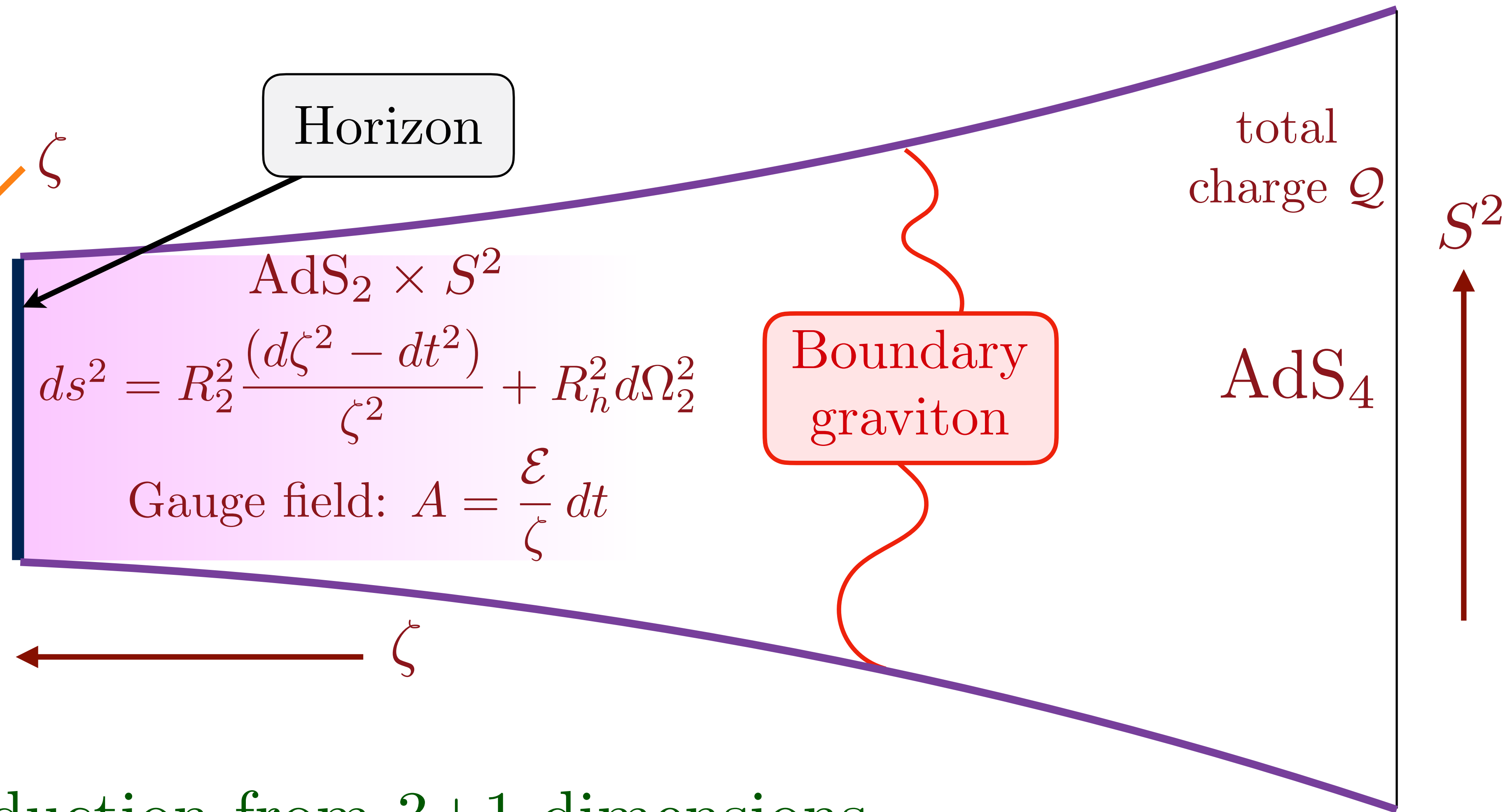
Sachdev PRL 2010



Reissner-Nordstrom black hole of Einstein-Maxwell theory



Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Thermodynamics of quantum black holes with charge Q :



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 &= \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) \exp \left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)] \right)
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$$S(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} - \frac{3}{4} \ln \left(\frac{\hbar c^5}{GT^2} \right)$$

The $\ln T$ term is the SYK/boundary-graviton correction to Bekenstein-Hawking.

There is also a
 $-\frac{559}{180} \ln \left(\frac{A_0 c^3}{\hbar G} \right)$
 term from other massless
 modes; Sen (2011)
 Iliesiu, Murthy, Turiaci (2022)

Black hole questions and answers

Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

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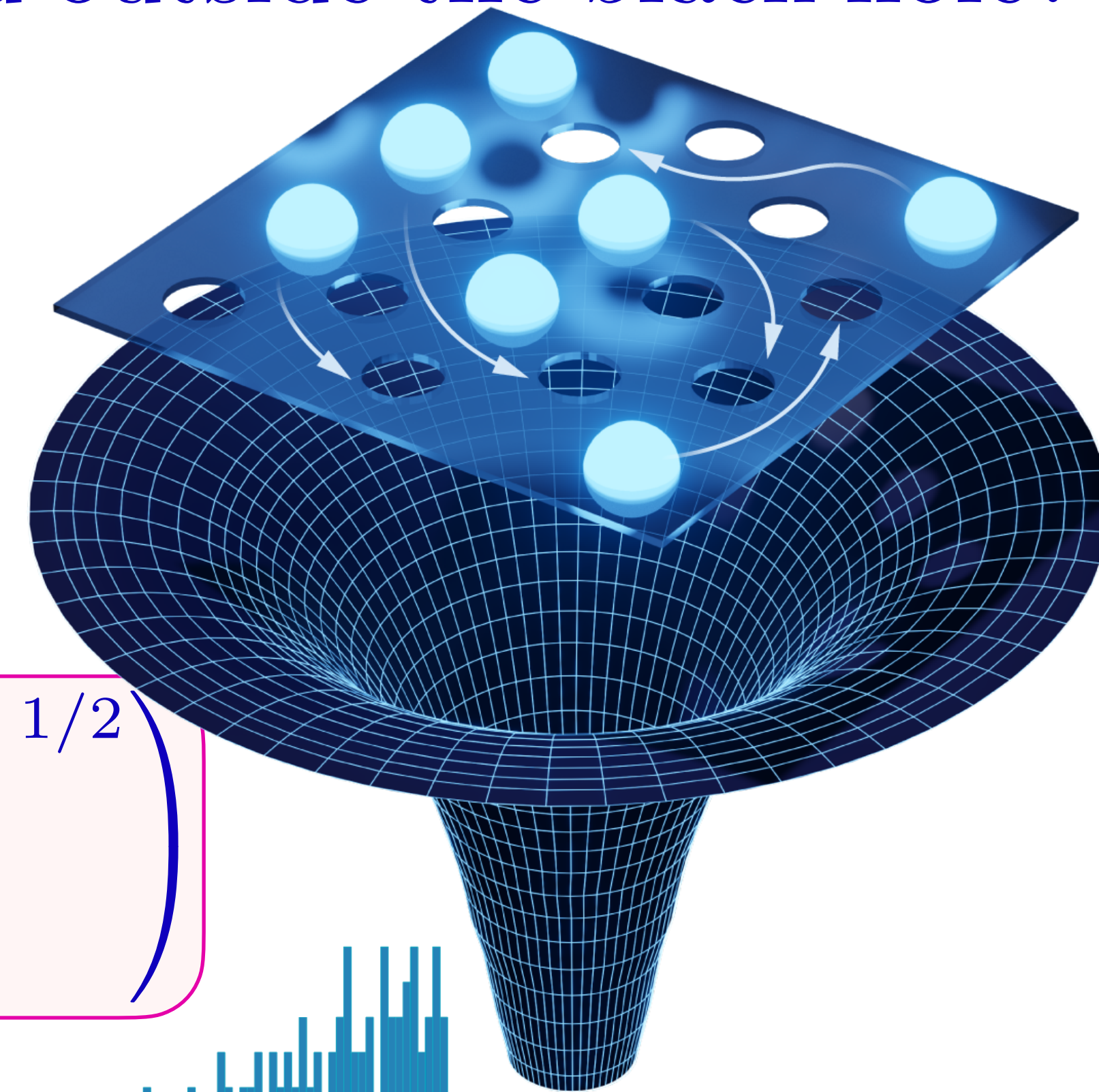
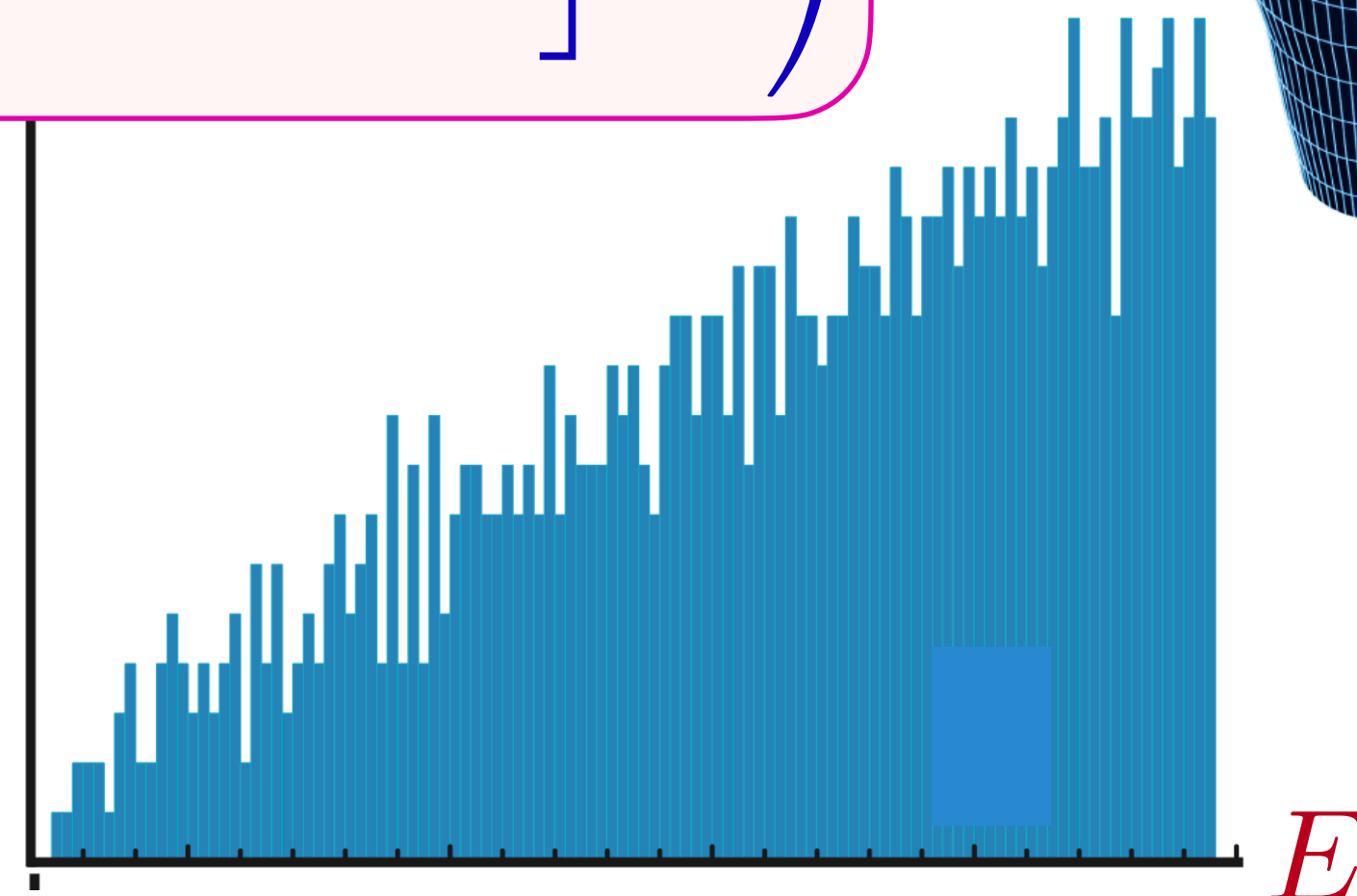
Yes, for charged black holes:

- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-559/180} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.

$D(E)$



Black hole questions and answers

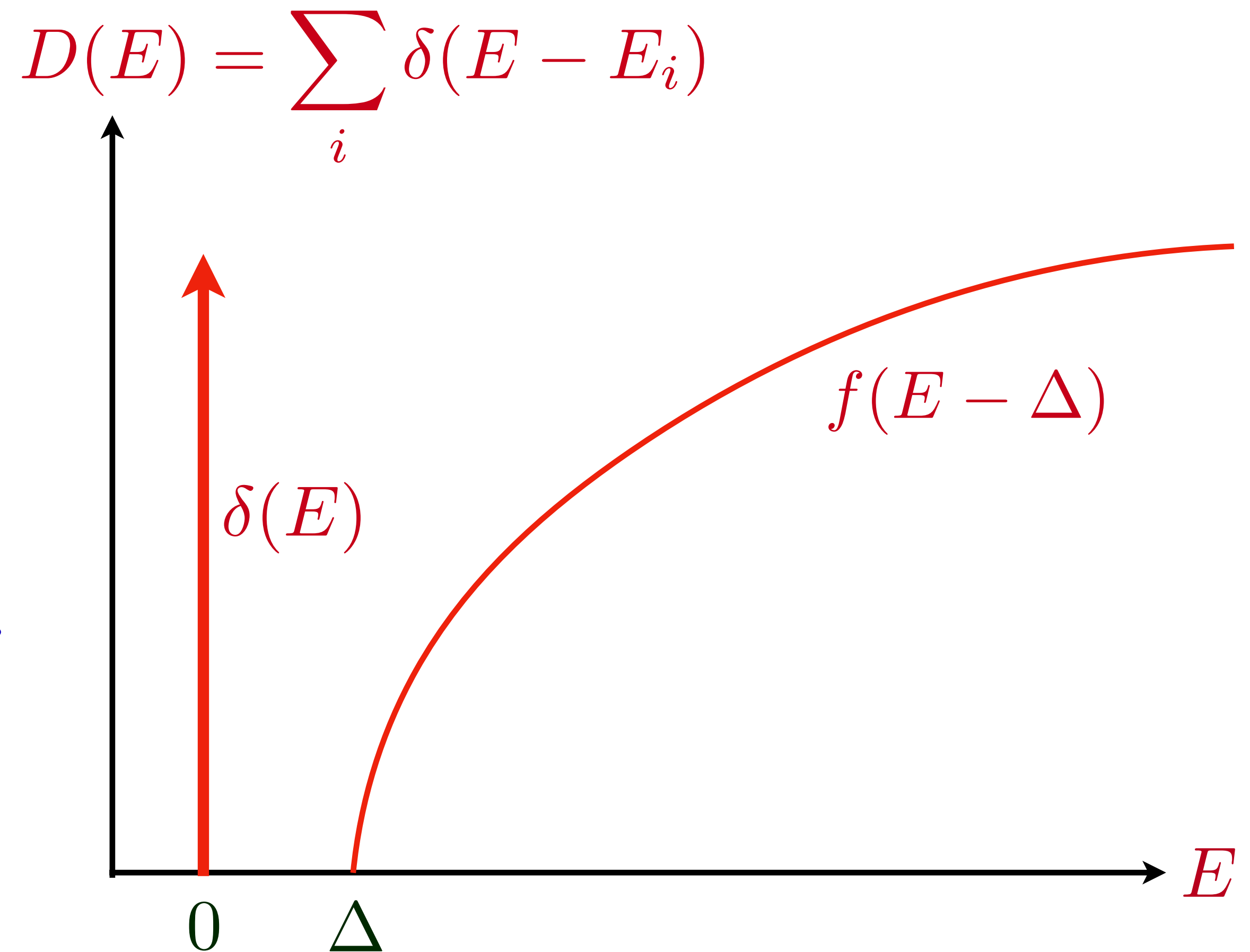
Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \delta(E) + \theta(E - \Delta) f(E - \Delta) + \dots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020

L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

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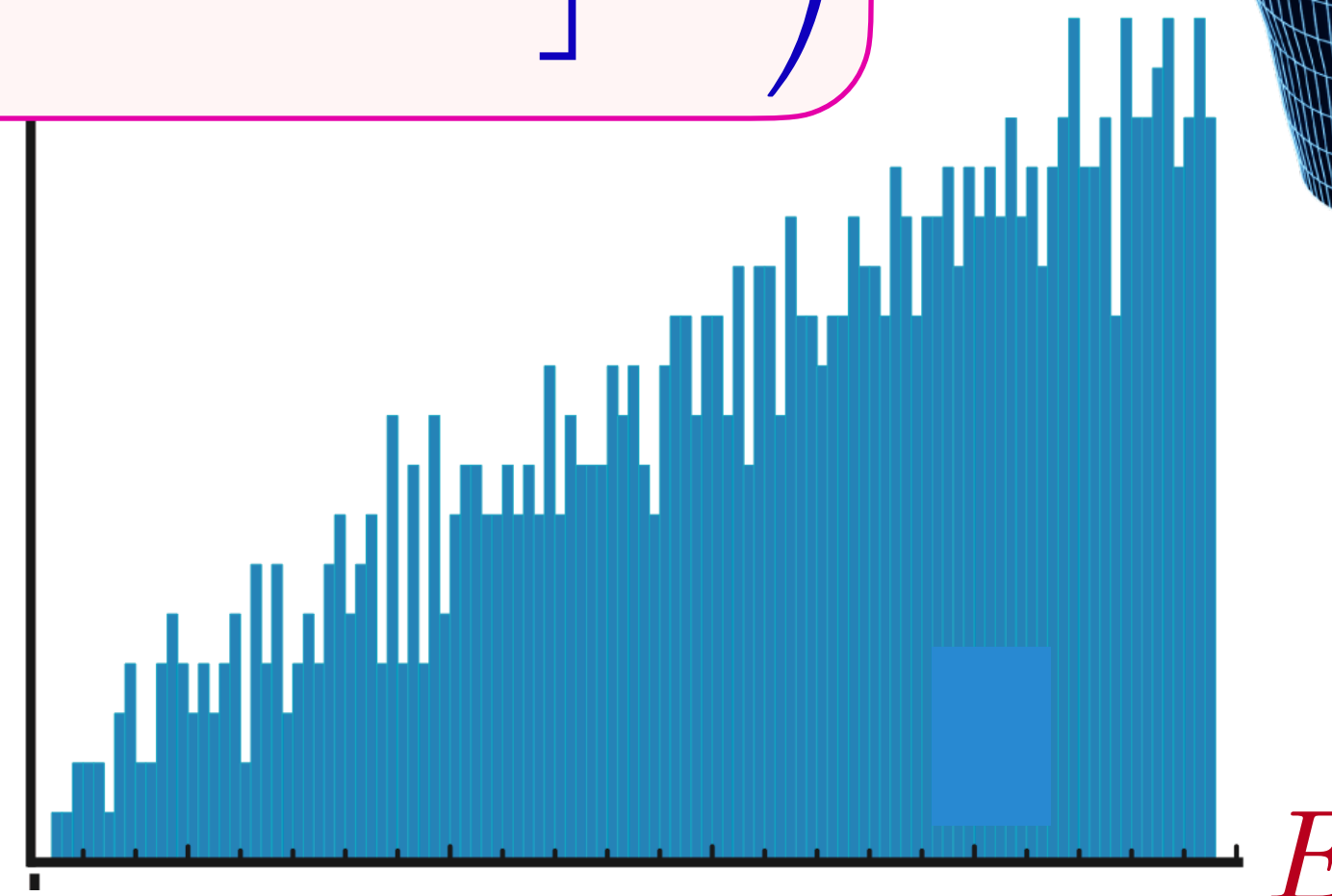
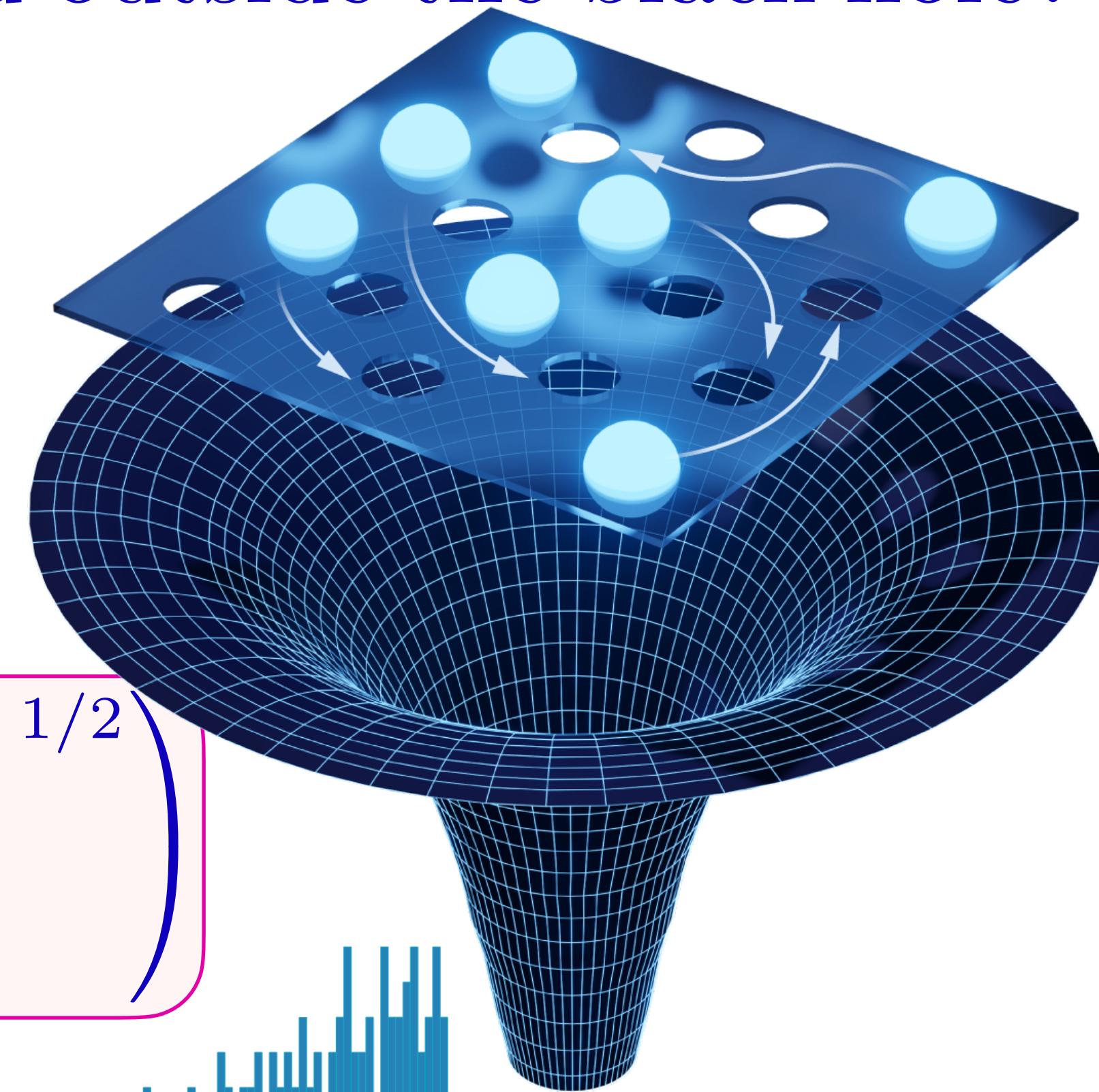
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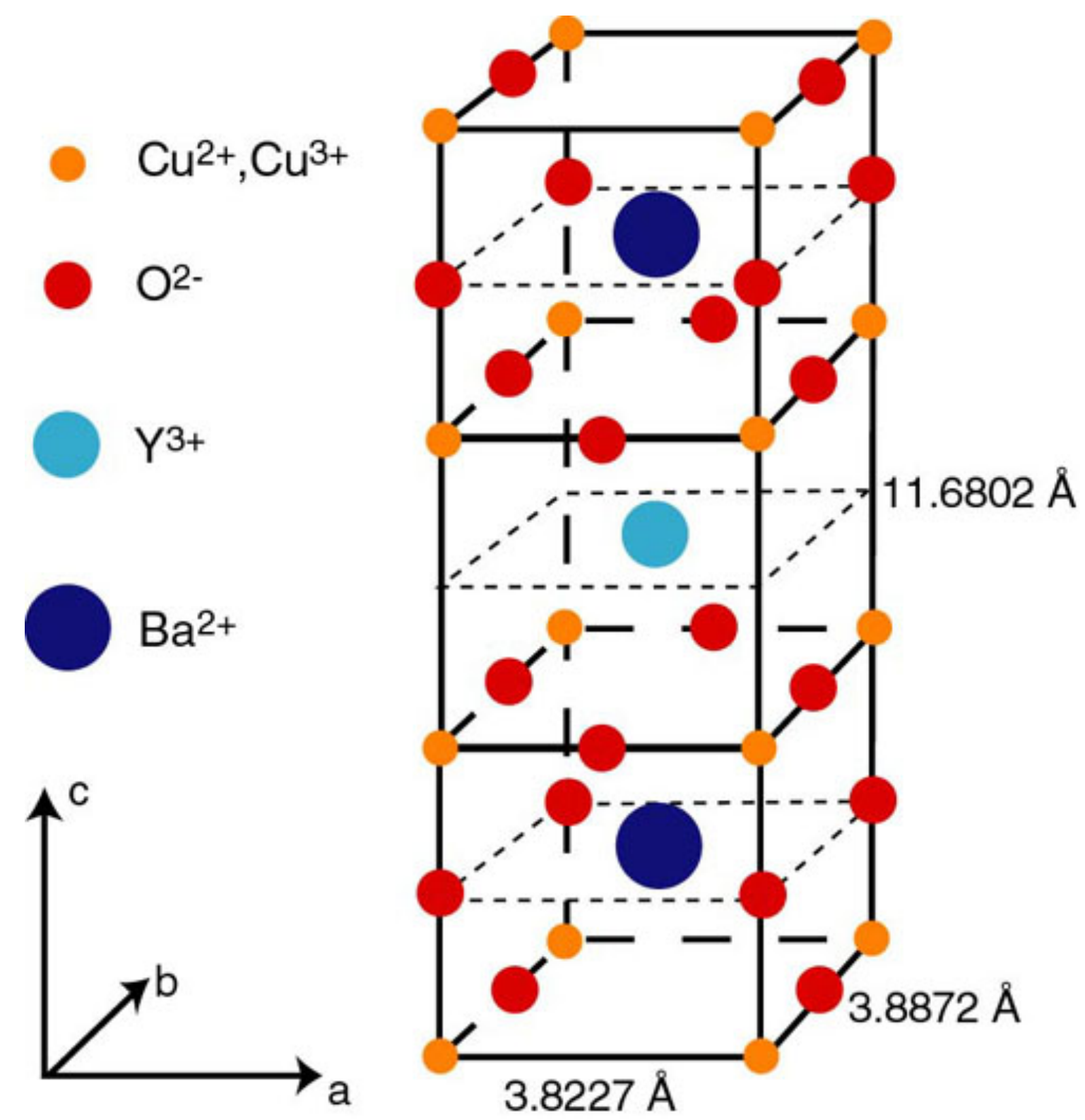
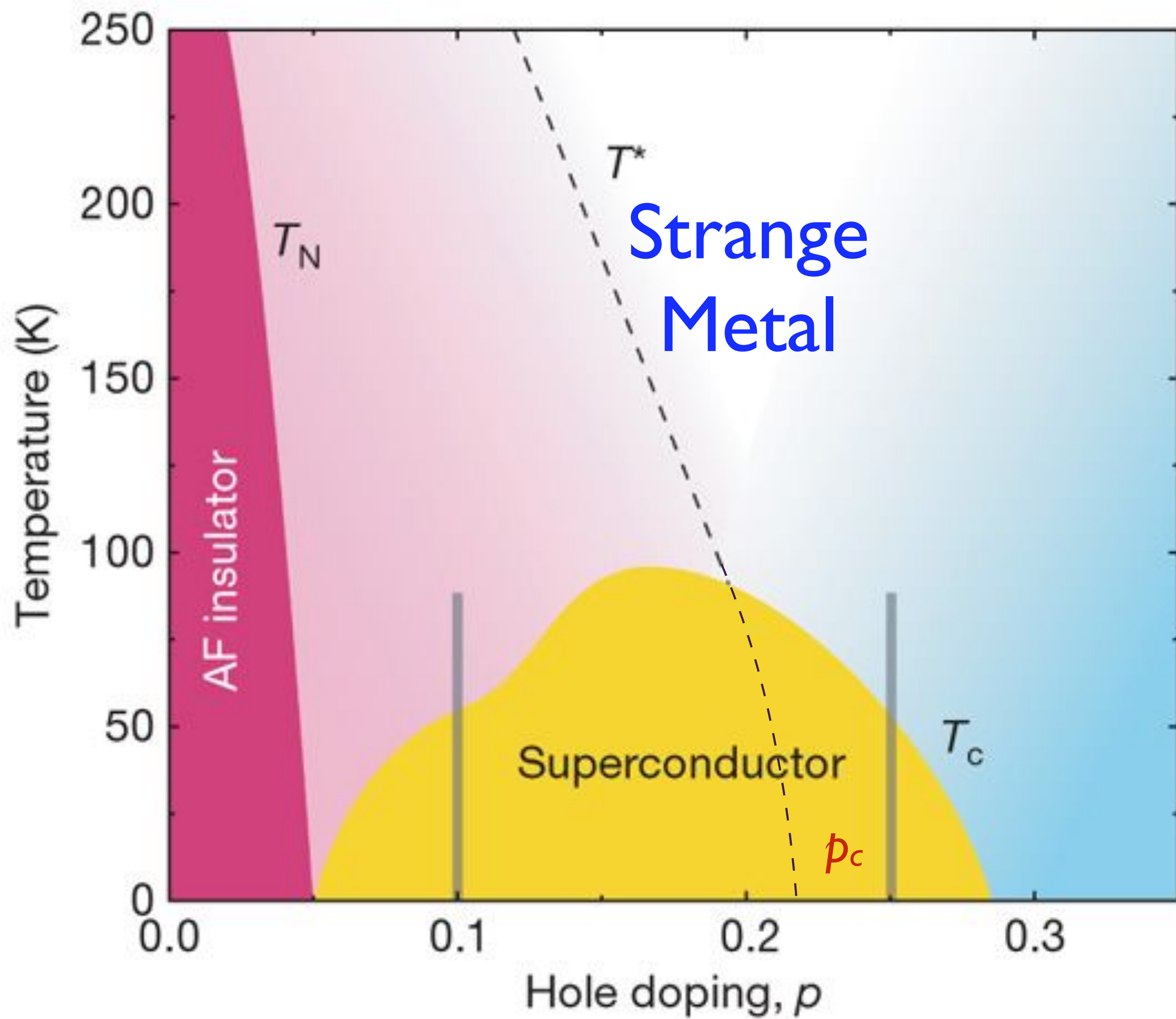
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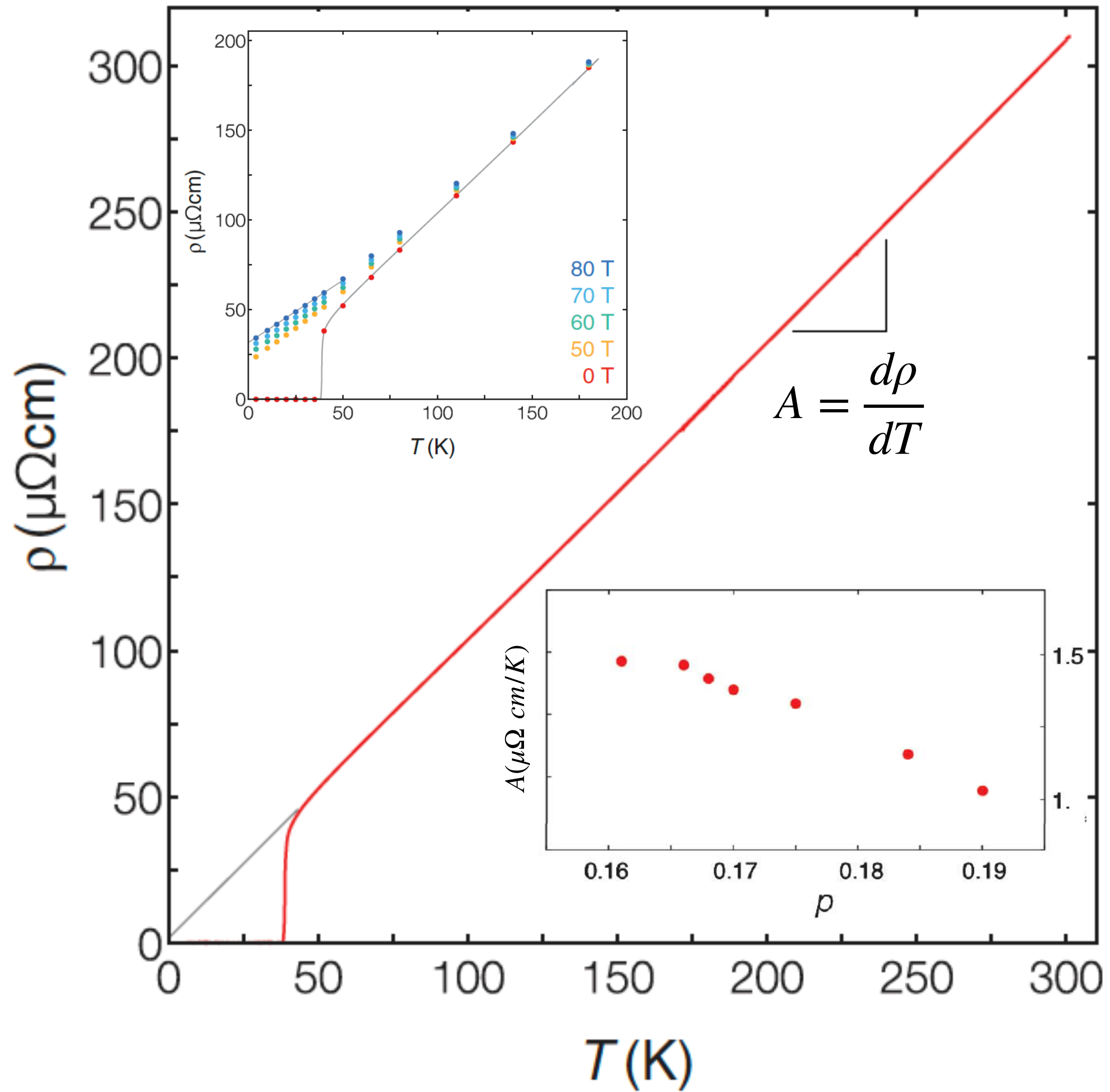
- ‘Wormhole’ contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.

Saad, Shenker, Stanford (2019)

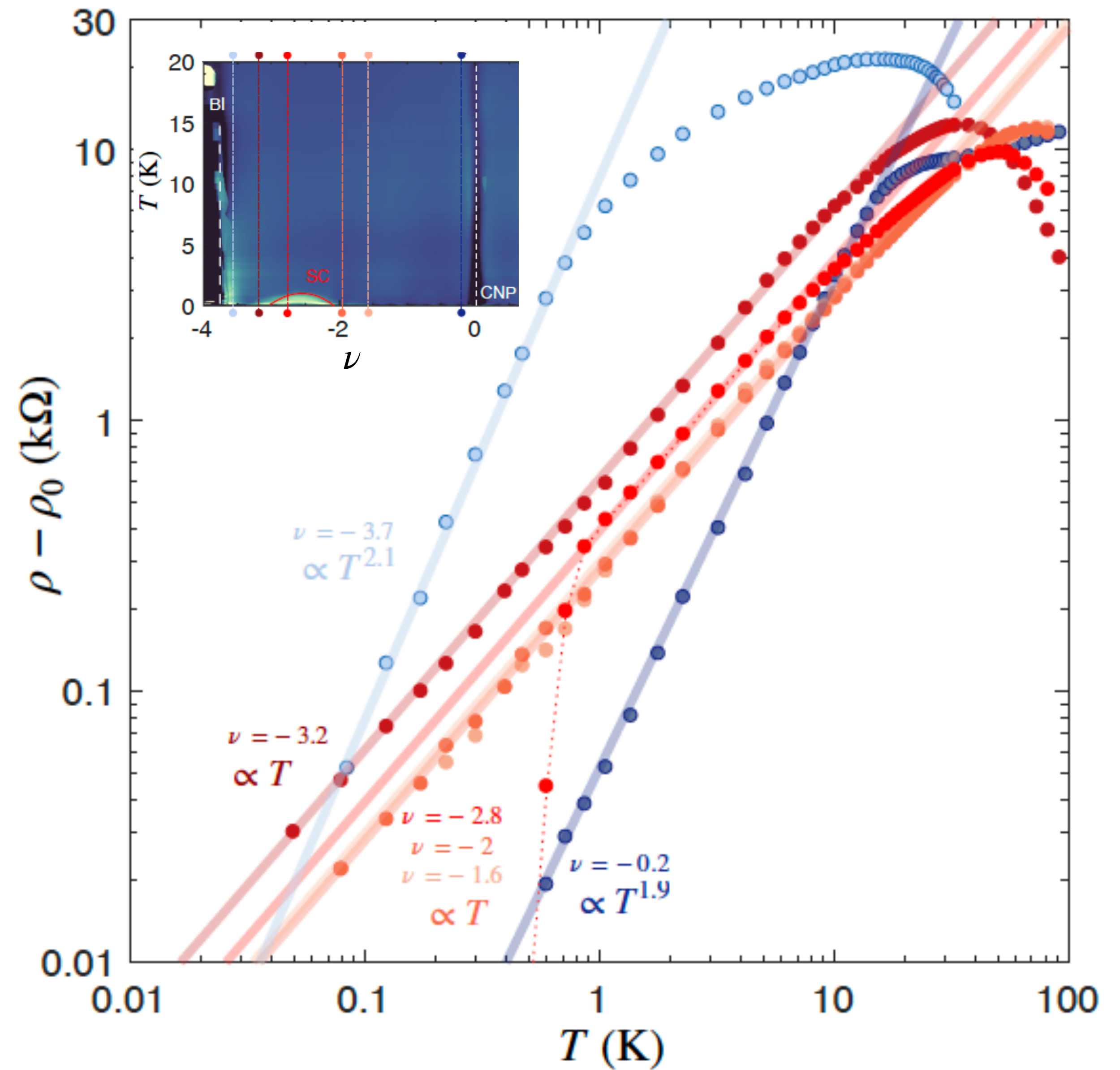


**Strange
metals**





LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).
Metals with $\rho(T) > h/e^2$ are bad metals.

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Metals with $\rho(T) > h/e^2$ are bad metals.
- Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

Properties of a strange metal:

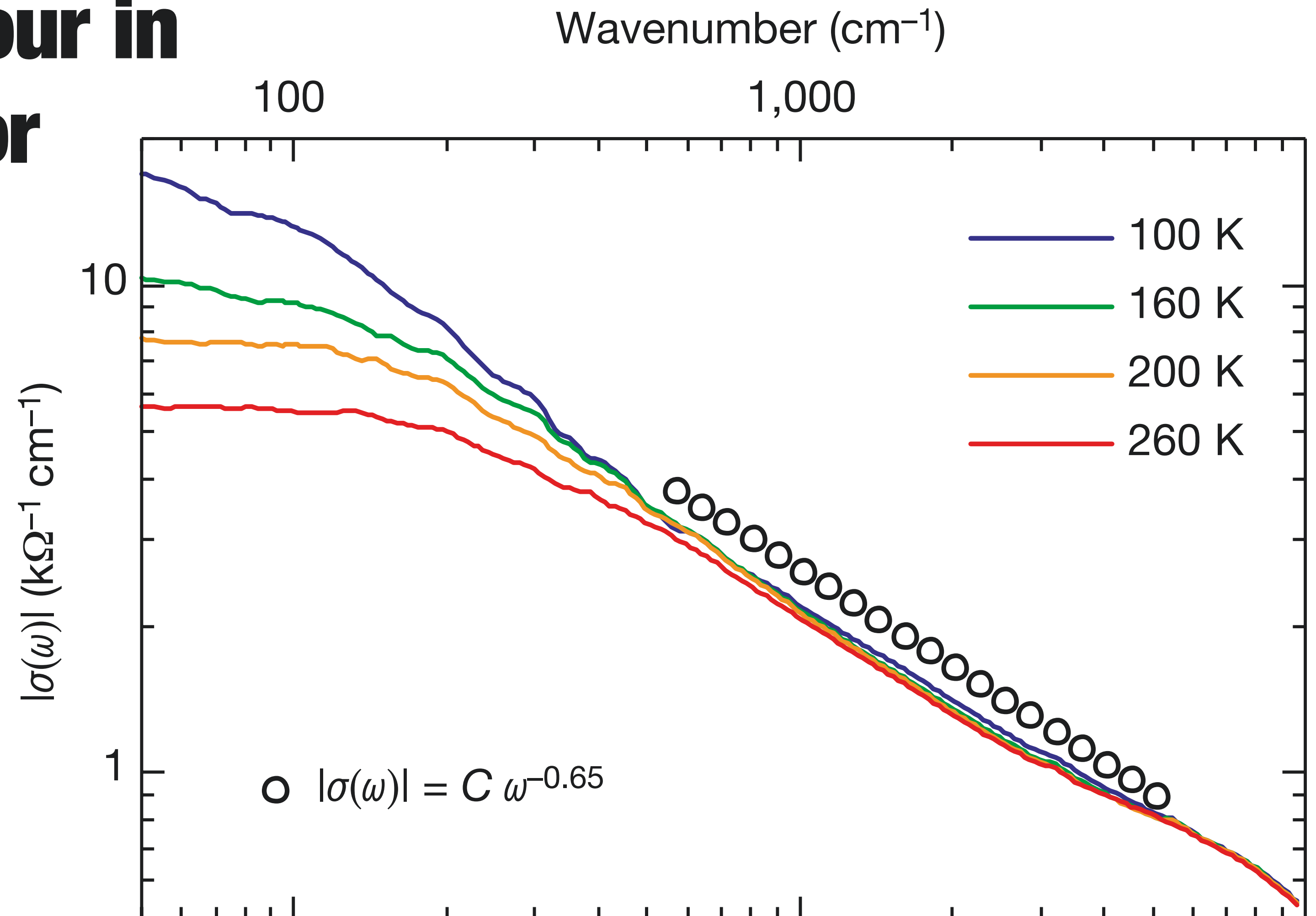
- Optical conductivity

Nature **425**, 271 (2003)

D. van der Marel^{1*}, H. J. A. Molegraaf^{1*}, J. Zaanen², Z. Nussinov^{2*},
F. Carbone^{1*}, A. Damascelli^{3*}, H. Eisaki^{3*}, M. Greven³, P. H. Kes² & M. Li²

Quantum critical behaviour in a high- T_c superconductor

But no $\hbar\omega/(k_B T)$ scaling.



- Optical conductivity

B. Michon,^{1,2,3} C. Berthod,³ C. W. Rischau,³ A. Ataei,⁴ L. Chen,⁴
S. Komiya,⁵ S. Ono,⁵ L. Taillefer,^{4,6} D. van der Marel,³ and A. Georges^{7,8,3}

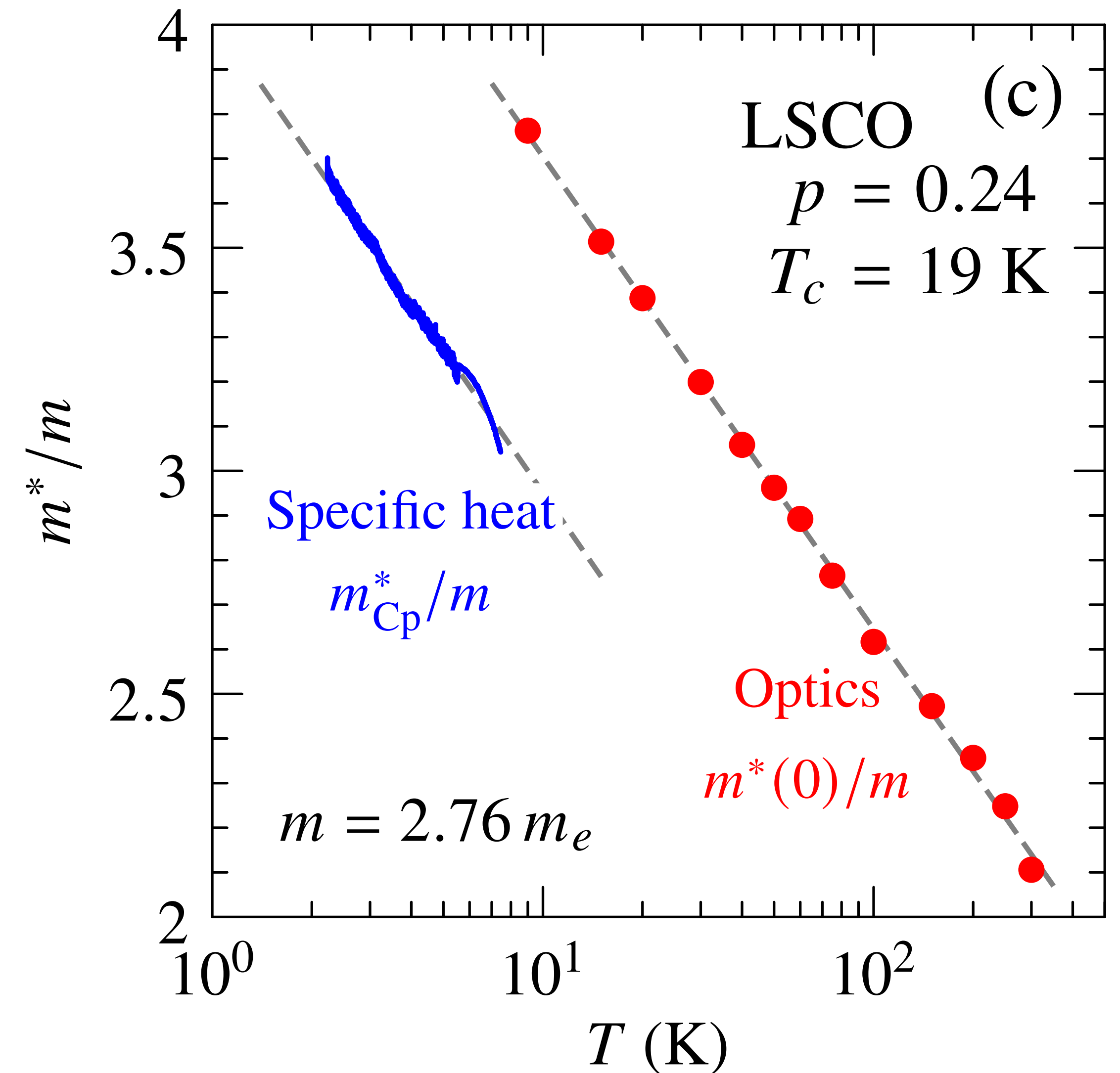
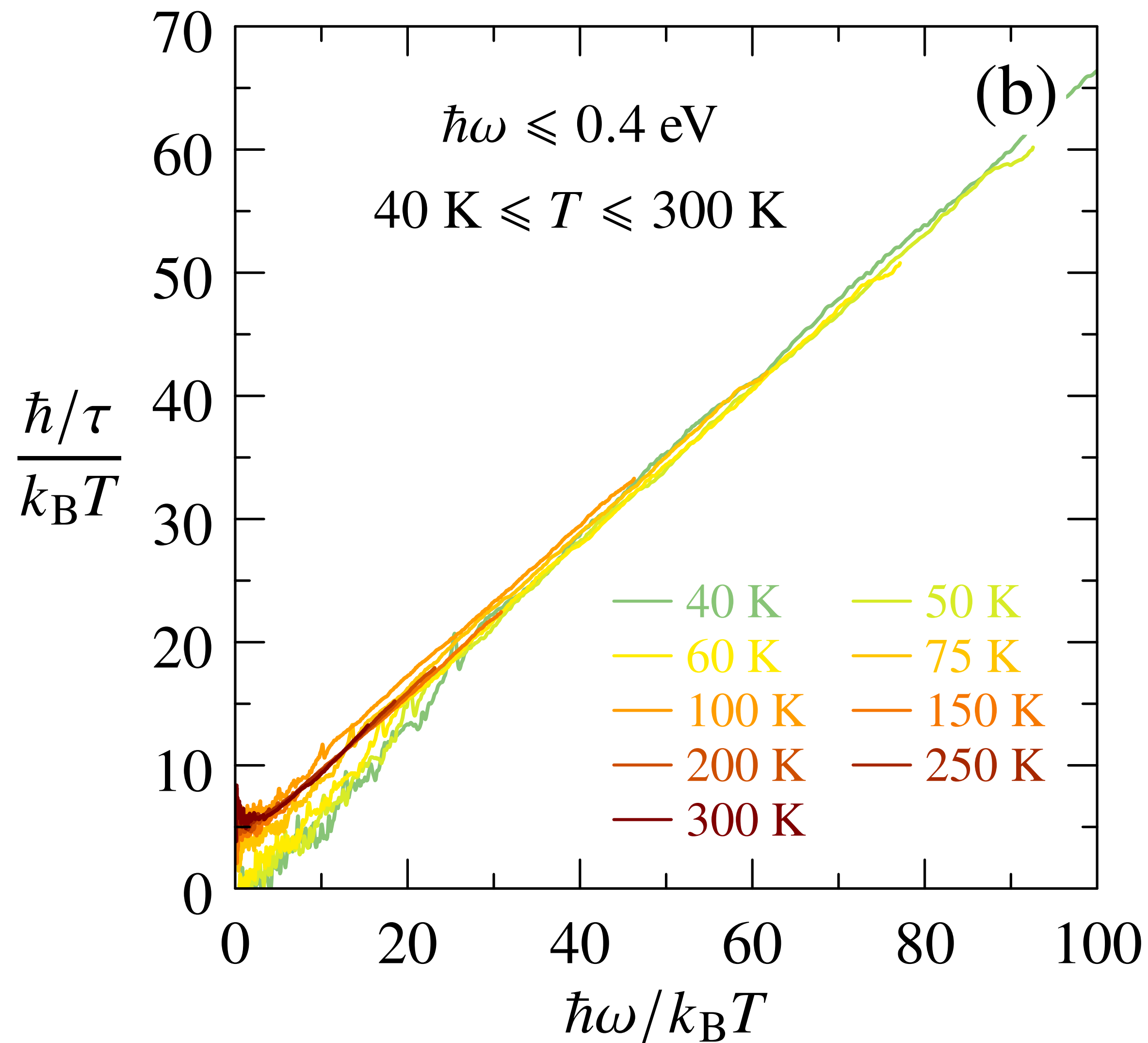
Planckian Behavior of Cuprate Superconductors: Reconciling the Scaling of Optical Conductivity with Resistivity and Specific Heat

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

$$\text{Causality: } \frac{m^*(\omega)}{m} \sim \ln \left(\frac{\Lambda}{\text{Max}(\hbar\omega, k_B T)} \right)$$

● Optical conductivity

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Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).

Metals with $\rho(T) > h/e^2$ are bad metals.

- Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

- Optical conductivity

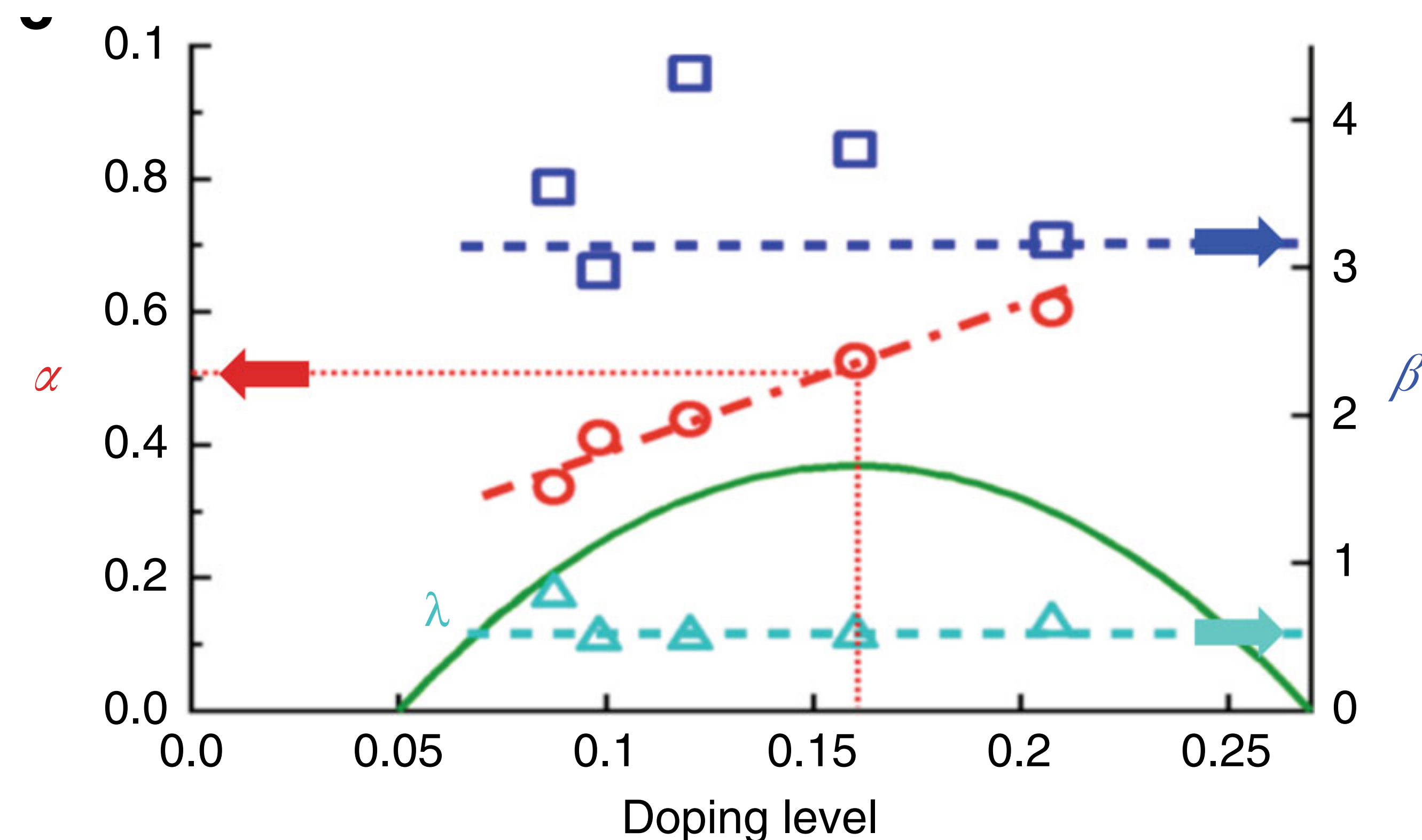
$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon.....A. Georges, arXiv:2205.04030

● Photoemission

A unified form of low-energy nodal electronic interactions in hole-doped cuprate superconductors

T.J. Reber^{1,5*}, X. Zhou^{1*}, N.C. Plumb^{1,6}, S. Parham¹, J.A. Waugh¹, Y. Cao¹, Z. Sun^{1,7}, H. Li¹, Q. Wang¹, J.S. Wen², Z.J. Xu², G. Gu², Y. Yoshida³, H. Eisaki³, G.B. Arnold¹ & D.S. Dessau^{1,4*}



$$\Sigma''_{\text{PLL}}(\omega) = \Gamma_0 + \lambda \frac{[(\hbar\omega)^2 + (\beta k_B T)^2]^\alpha}{(\hbar\omega_N)^{2\alpha-1}}$$

Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).

Metals with $\rho(T) > h/e^2$ are bad metals.

- Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

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$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon.....A. Georges, arXiv:2205.04030

- Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad \text{T.J. Reber....D. Dessau, Nature Communications } \mathbf{10}, 5737 \text{ (2019)}$$

From
Yukawa-SYK models
to a universal theory of
strange metals



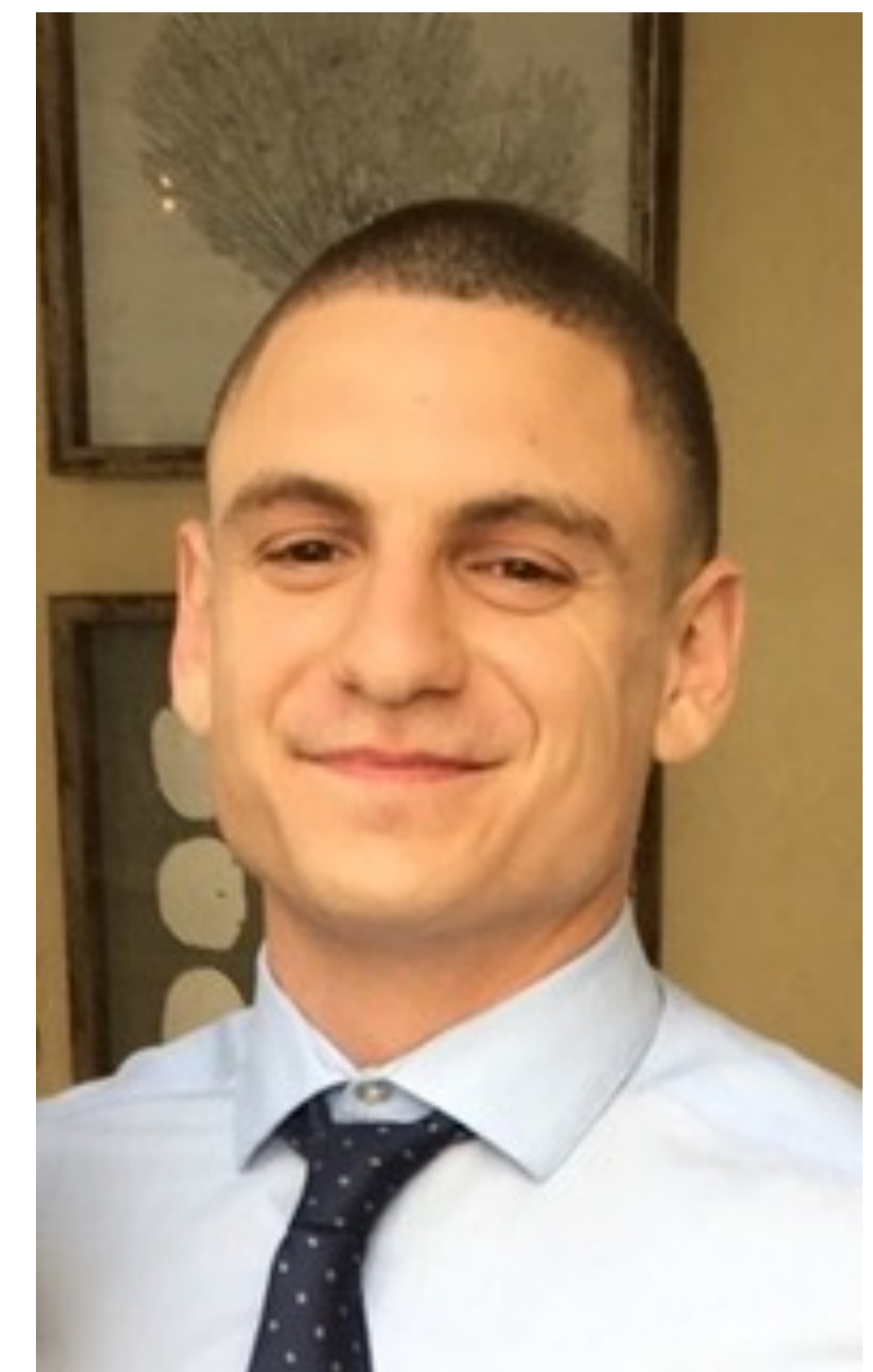
Aavishkar Patel

Flatiron Institute, NYC



Haoyu Guo

Harvard



Ilya Esterlis

Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)

A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)

E. Marcus and S. Vandoren, JHEP 01, 166 (2018)

Yuxuan Wang, PRL **124**, 017002 (2020)

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

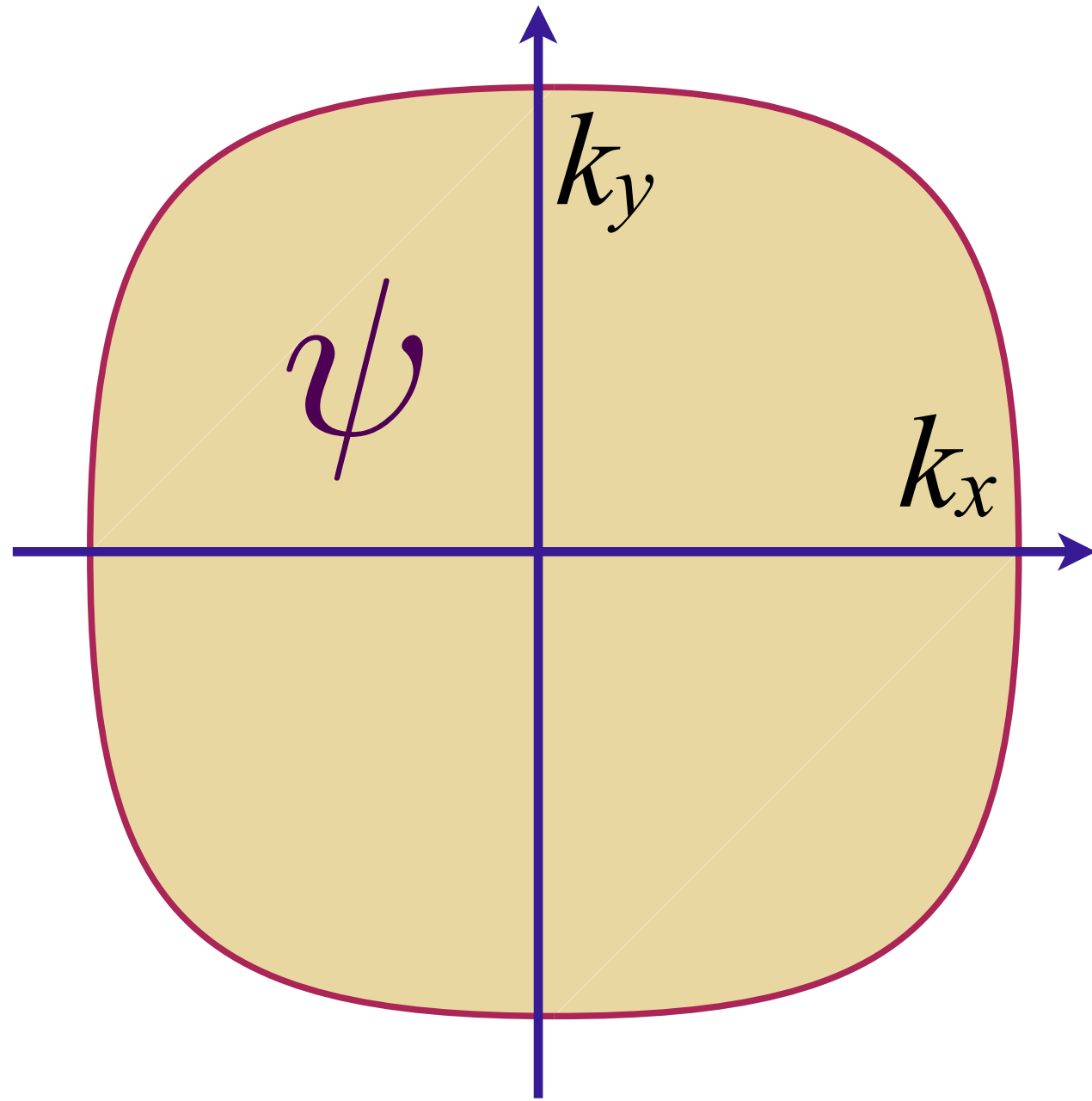
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)

W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Fermi surface

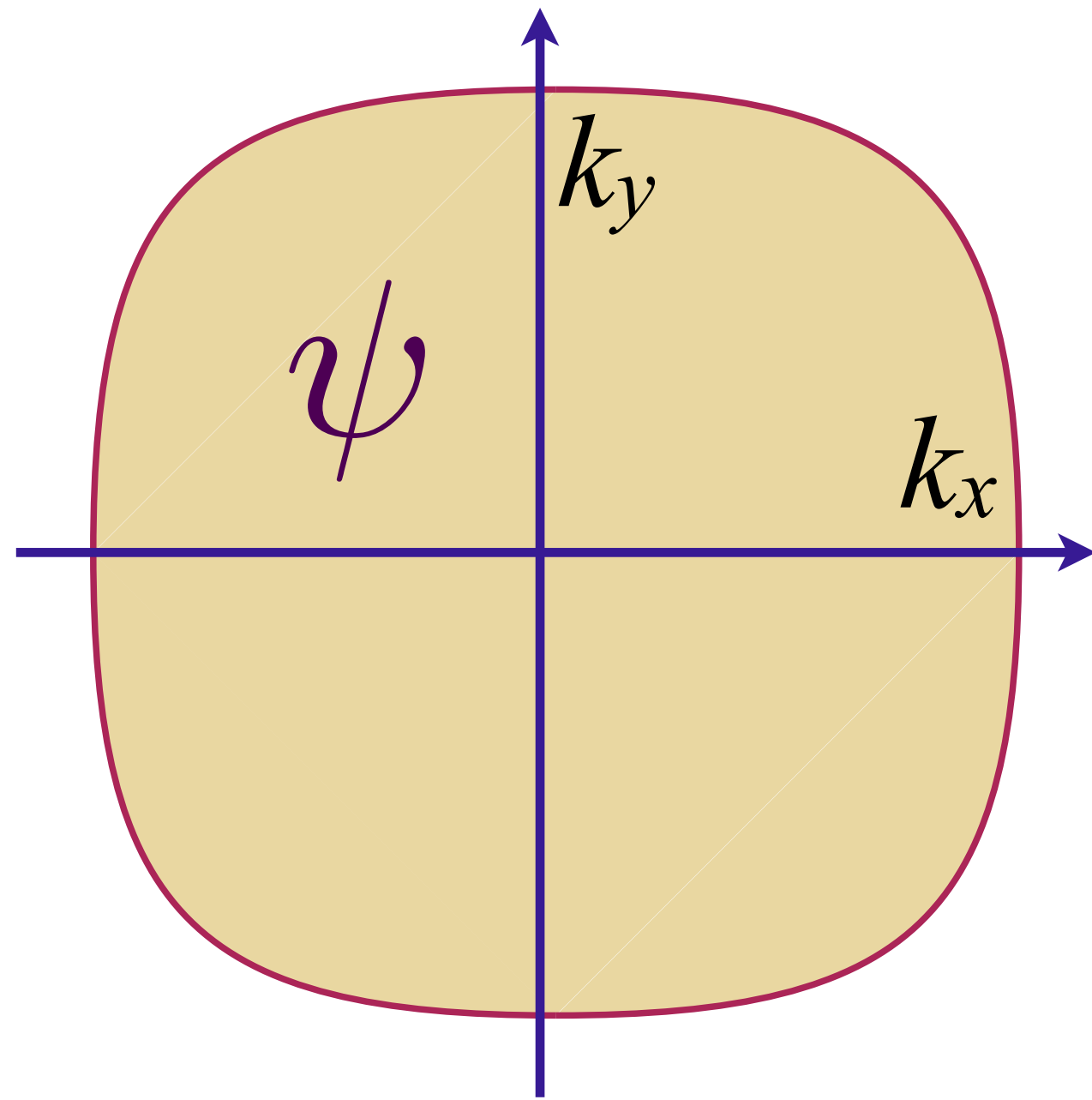
$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



$$-J \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \psi(\mathbf{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



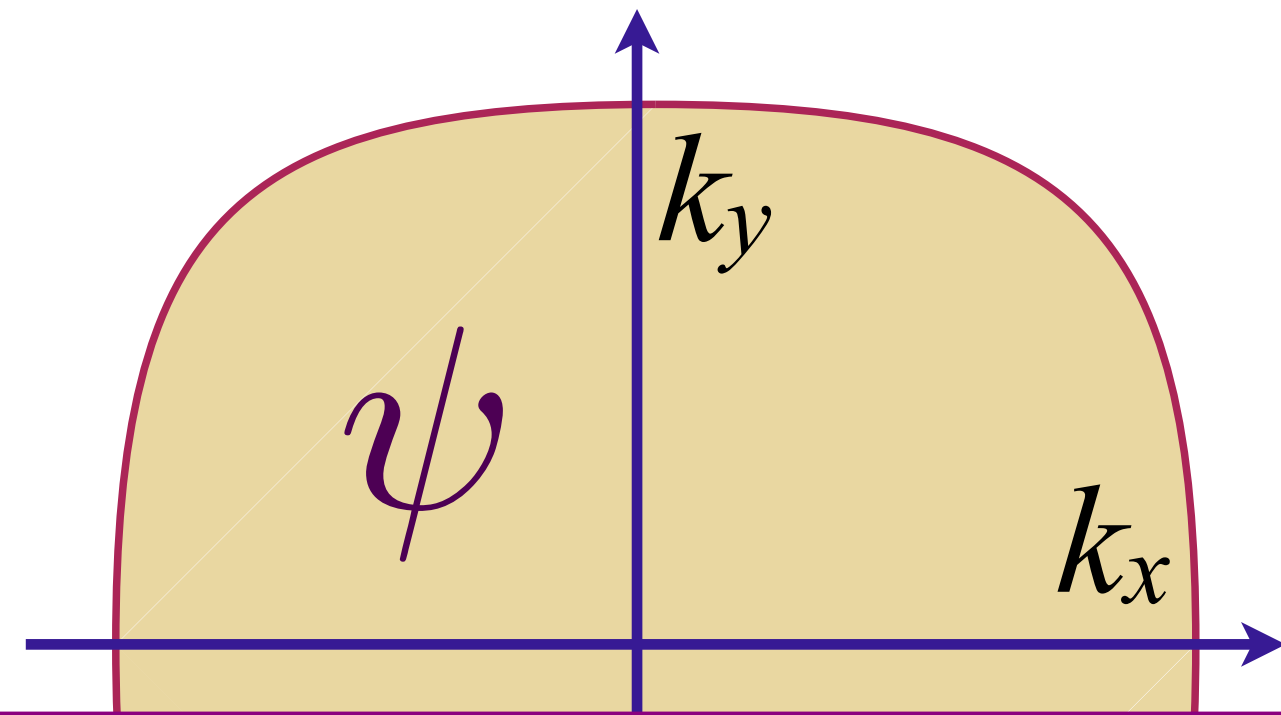
a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Solve in a large N limit with Yukawa coupling

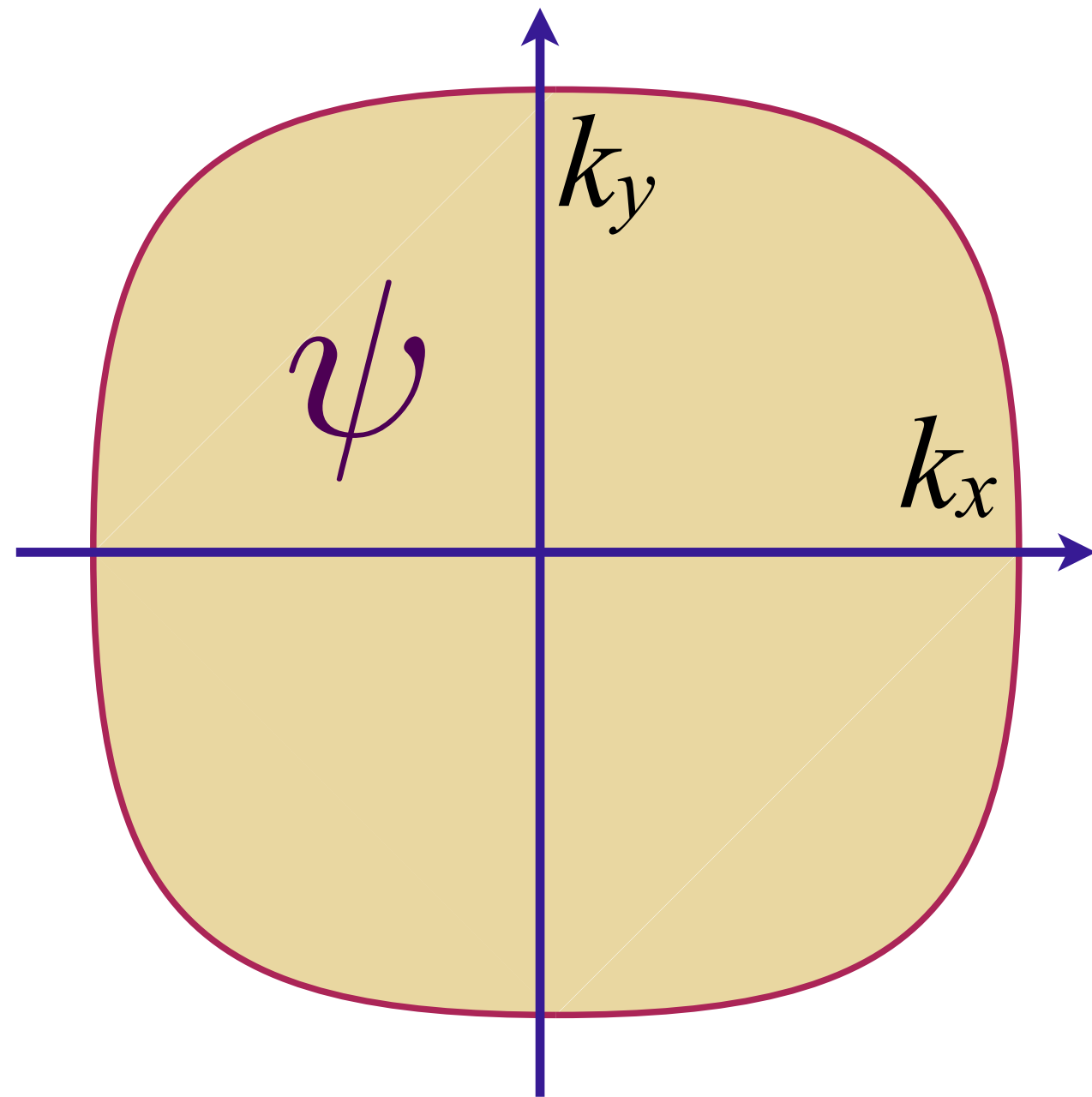
$$\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω :

$$\Sigma(\hat{\mathbf{k}}, i\omega) \sim -i \text{sgn}(\omega) |\omega|^{2/3} \quad , \quad G(\mathbf{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) - \Sigma(\hat{\mathbf{k}}, i\omega)} \quad , \quad D(\mathbf{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\text{Re} [\sigma(\omega)] = D\delta(\omega) + \dots$$

a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)

D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)

S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)

A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil

arXiv:2204.07585

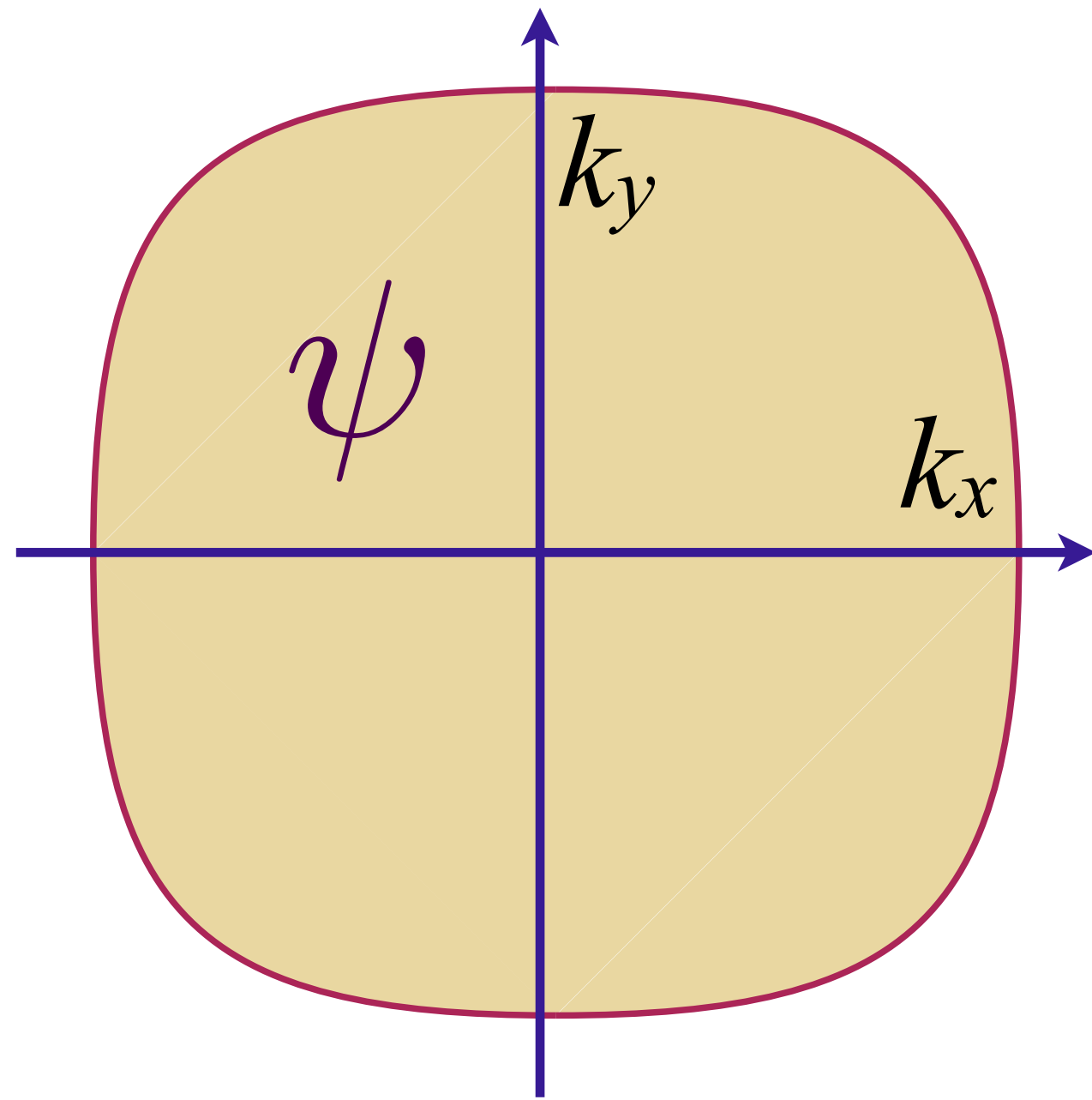
Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

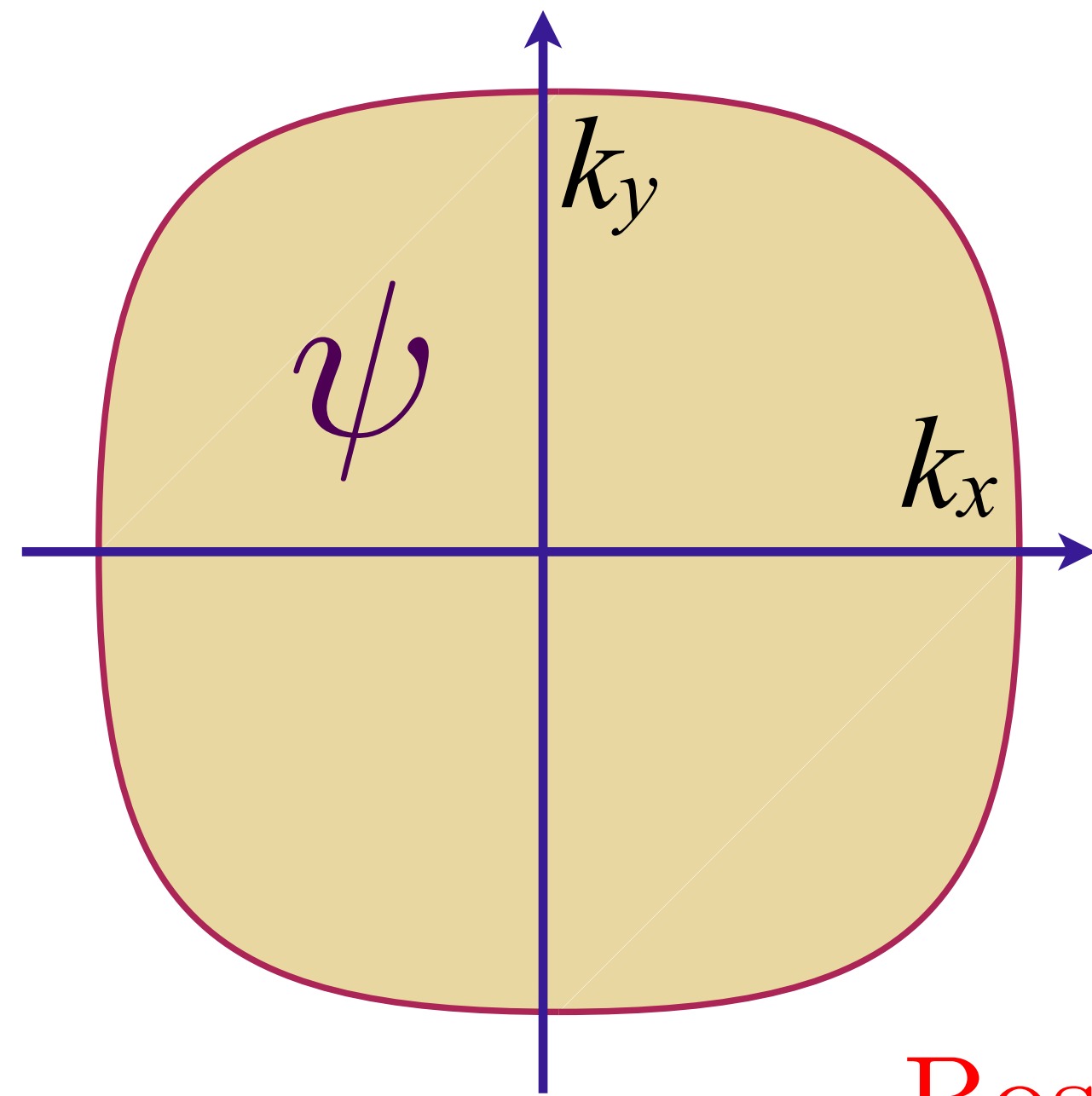
$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2\delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

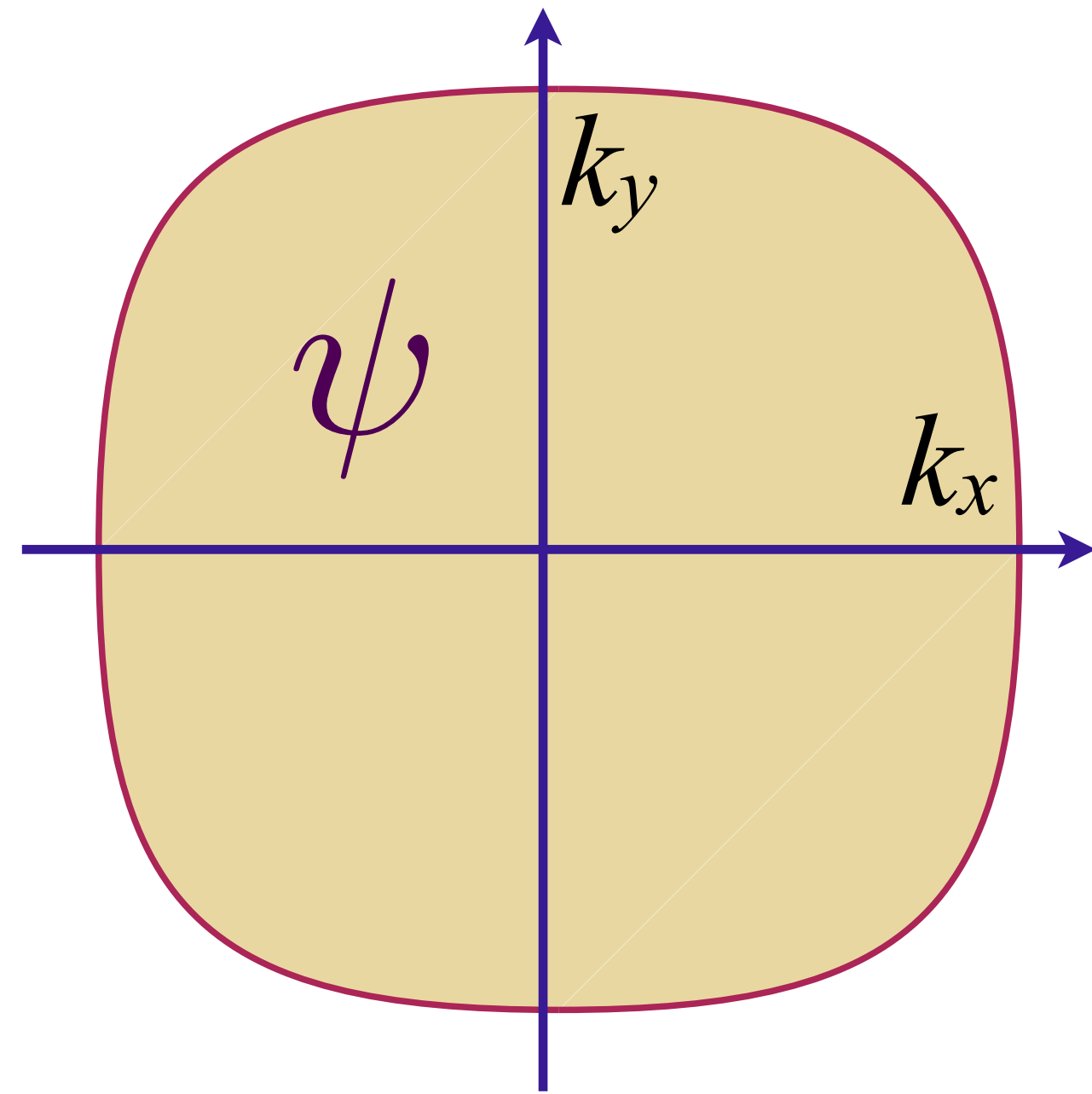
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|)$; $\frac{1}{\tau(\varepsilon)} \sim |\varepsilon|$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

But resistivity and optical conductivity are like a Fermi liquid,
with residual resistivity $\sim 1/v^2$.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NOT a strange metal

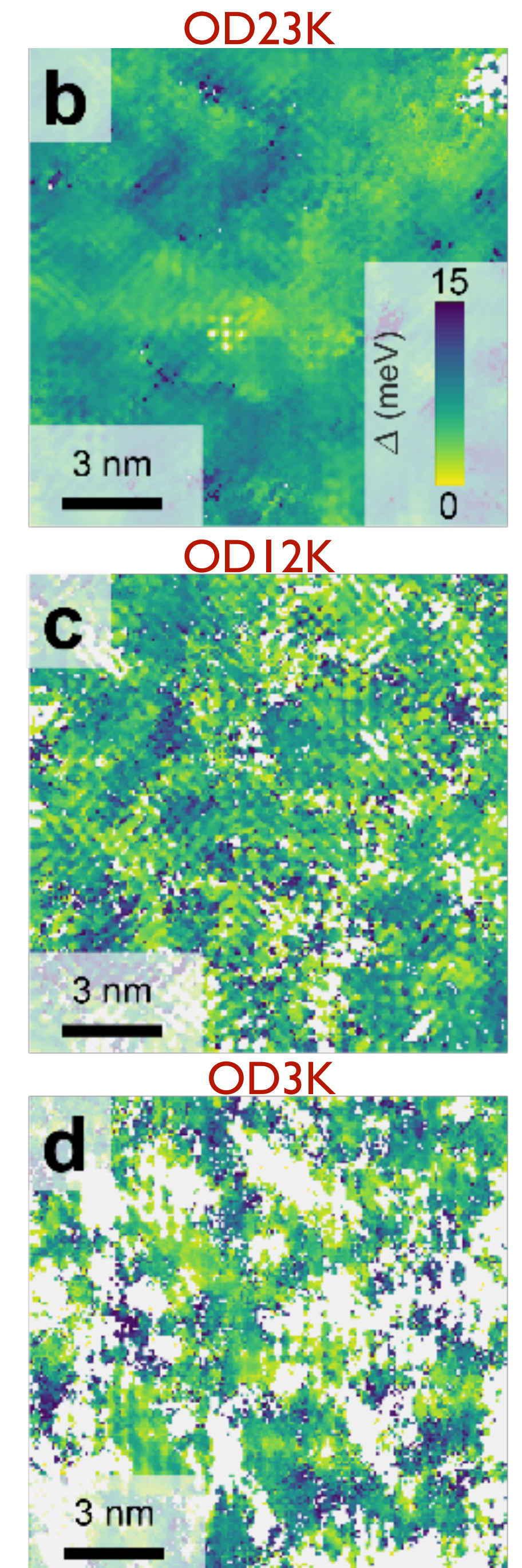
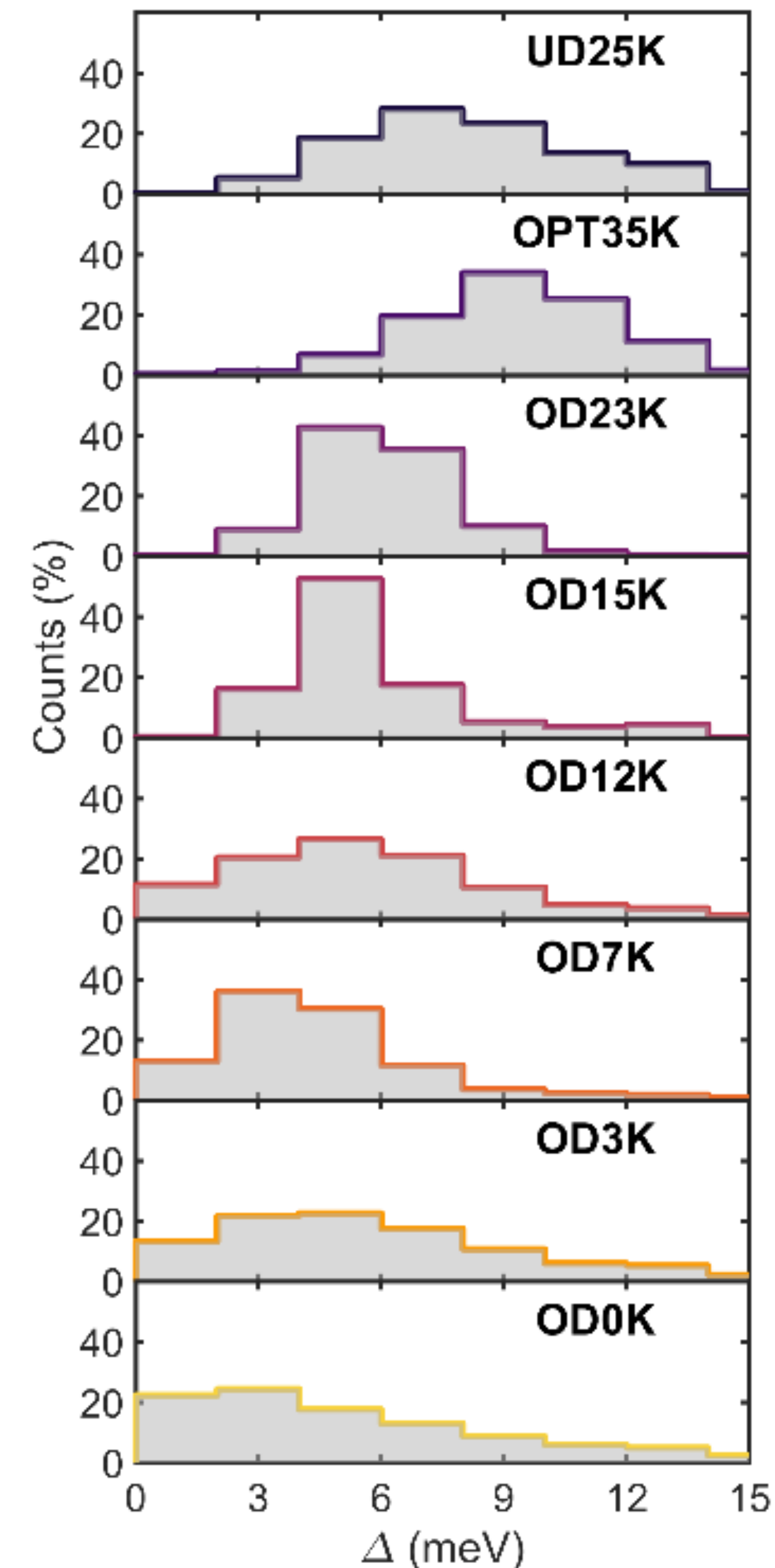
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(\text{Pb,Bi})_2\text{Sr}_2\text{CuO}_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(\text{Pb,Bi})_2\text{Sr}_2\text{CuO}_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

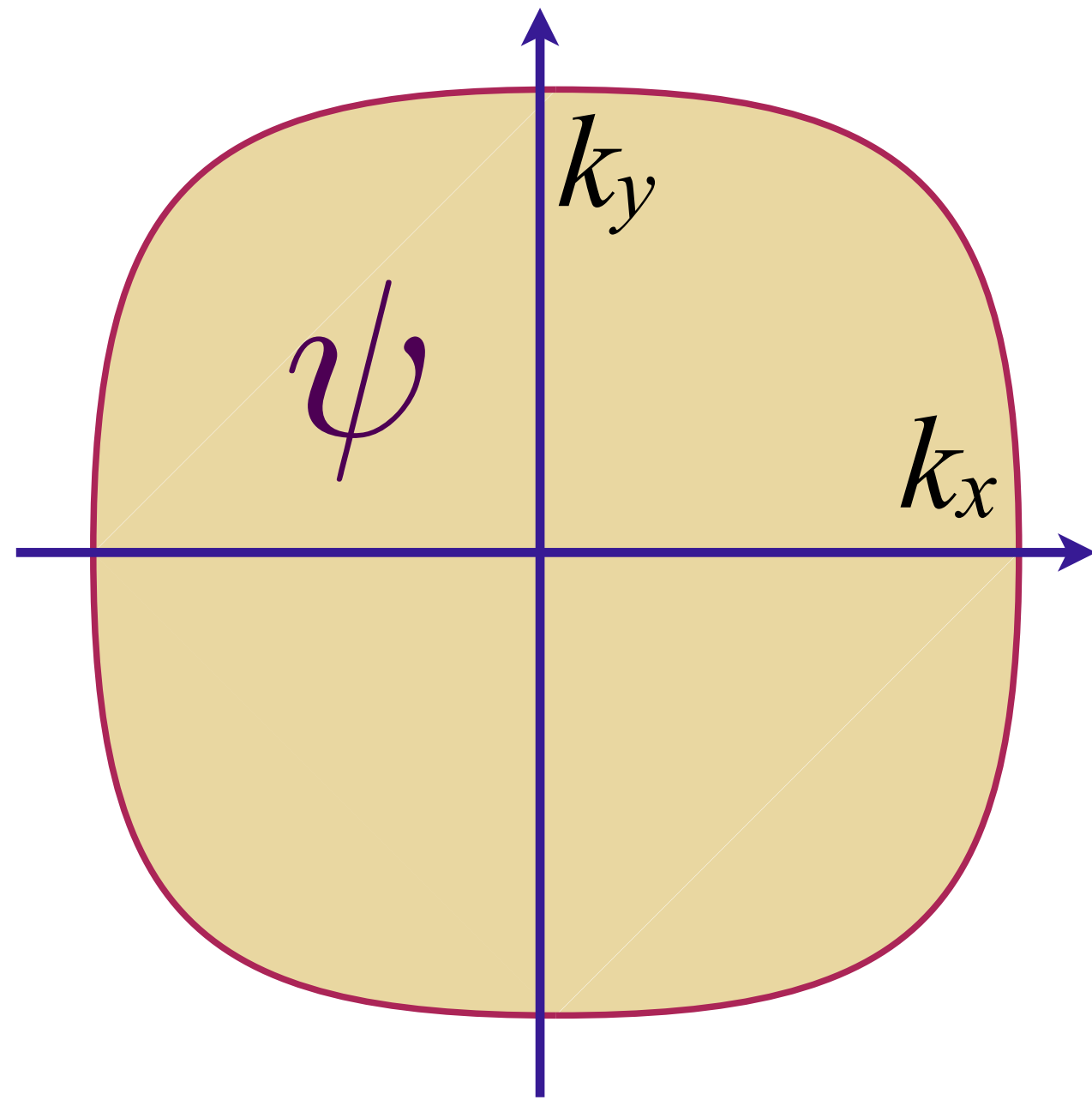
arXiv:2205.09740



Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

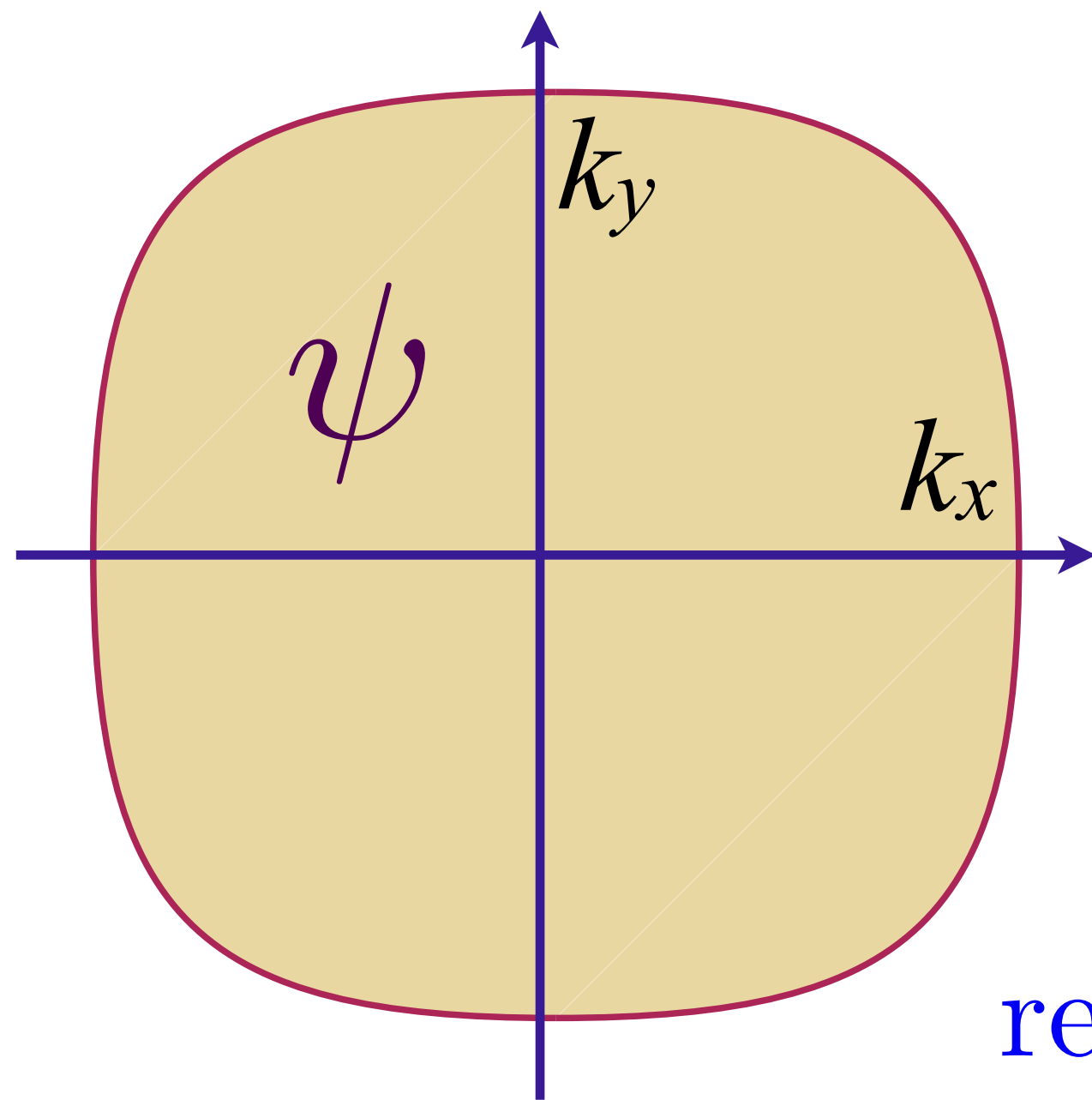


$$\frac{[\phi(\mathbf{r})]^2}{J + J'(\mathbf{r})} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Fermi surface coupled to a critical boson with disorder

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a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

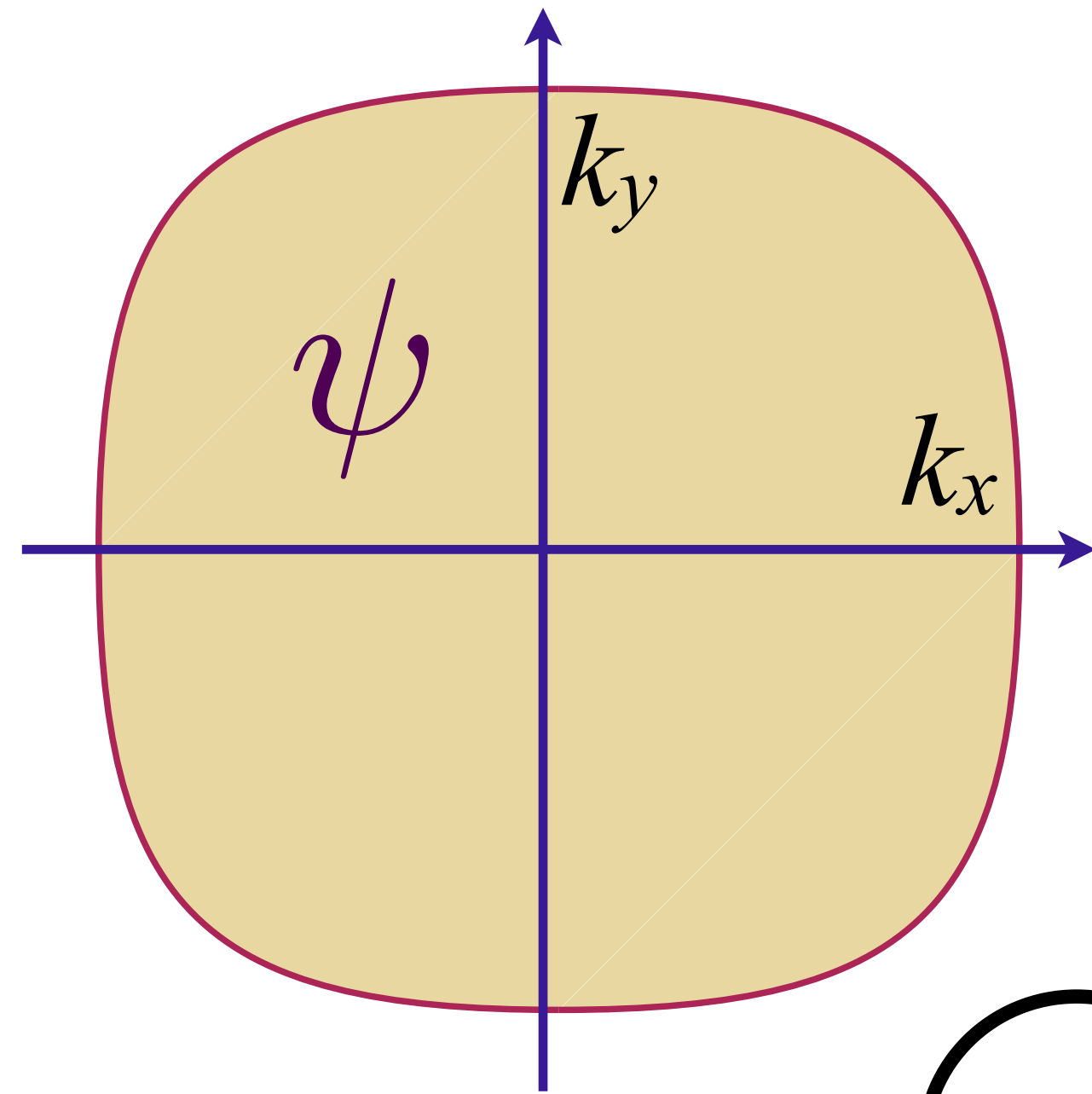
ϕ^2 “mass” disorder $J'(\mathbf{r})$ is strongly relevant;
 rescale ϕ to move disorder to the Yukawa coupling;
 can then be controlled in a SYK-like large N limit of ‘flavor’ indices,
 leading to a G - Σ - D - Π bi-local field theory.

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

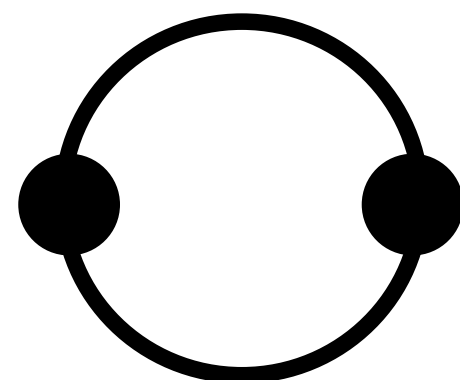
$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

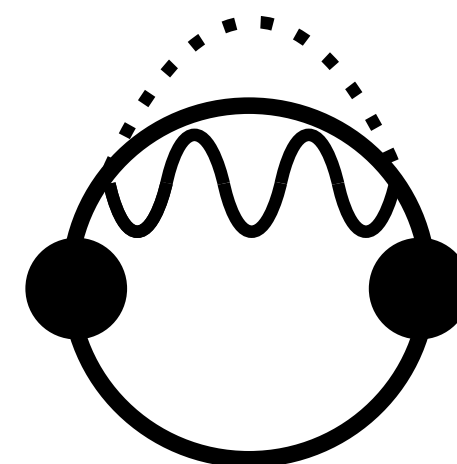


$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

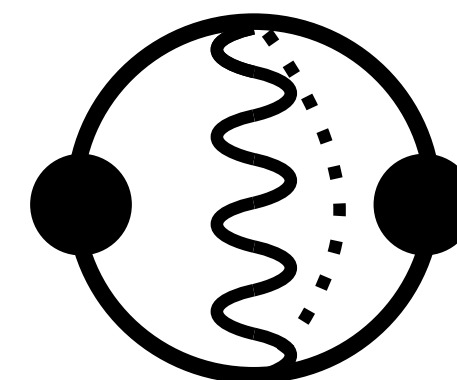
Conductivity:



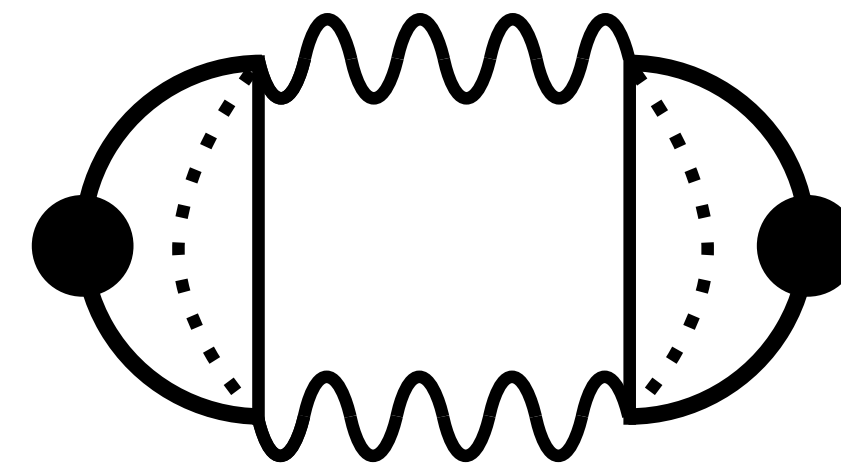
(a)
 σ_v



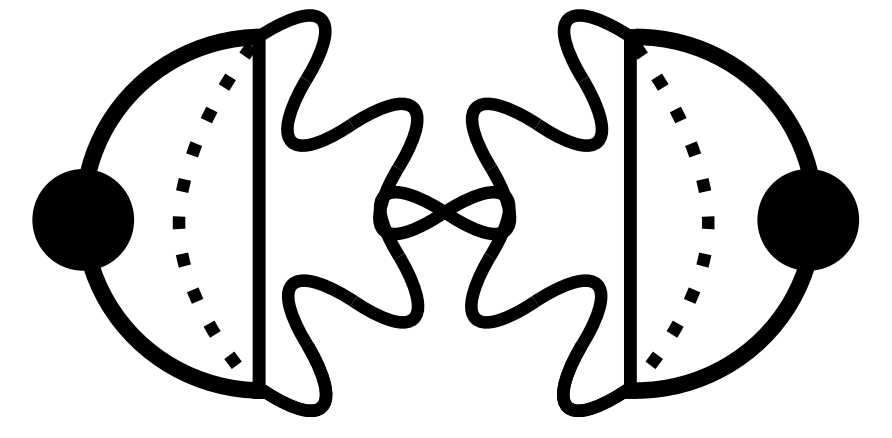
(b)
 $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$



(c)
 $\sigma_{V,g}$



(d)



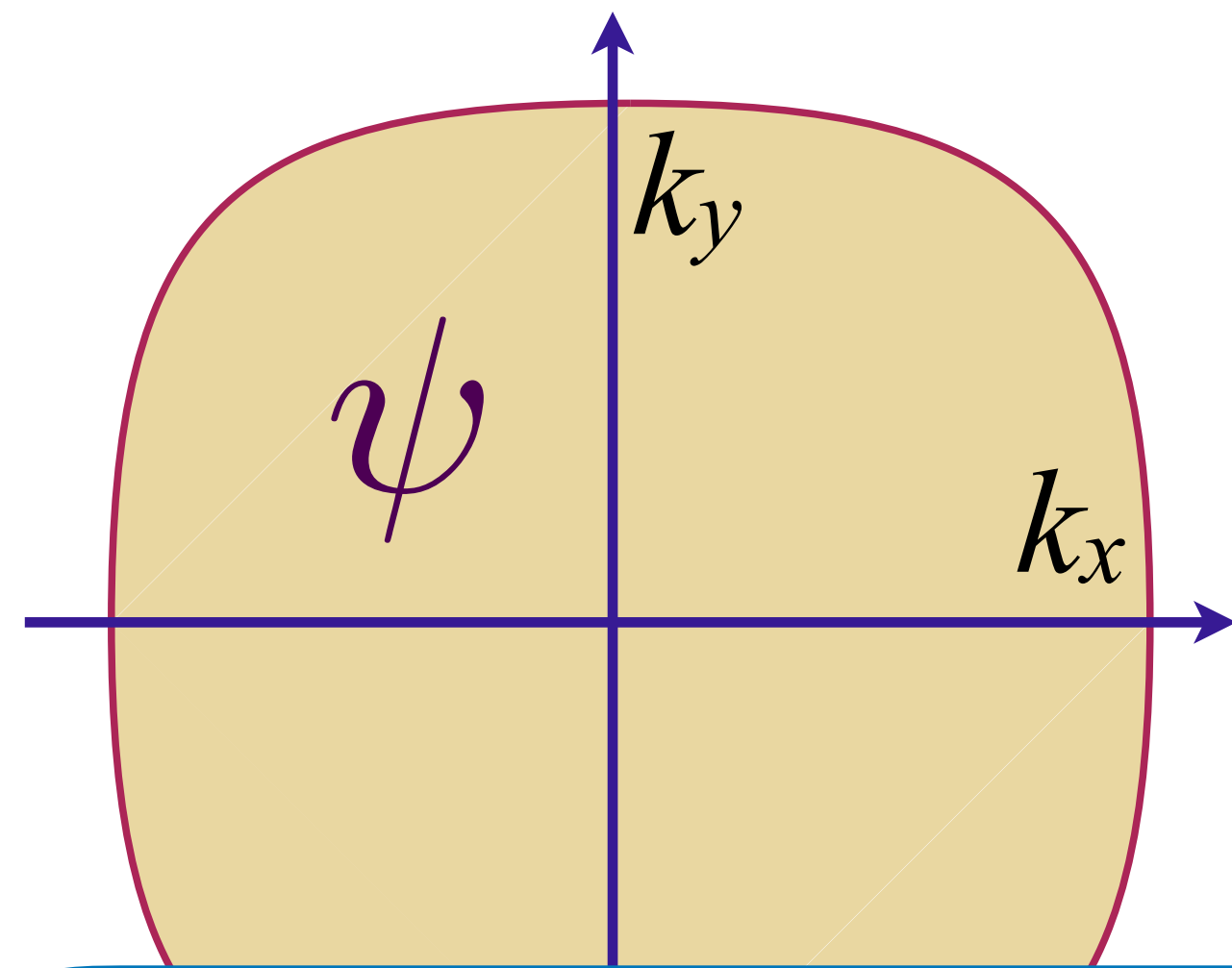
(e)

+ all ladders and bubbles.....

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega m^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ;
 Transport insensitive to g ; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Interaction disorder g'

A marginal Fermi liquid AND a strange metal

Summary

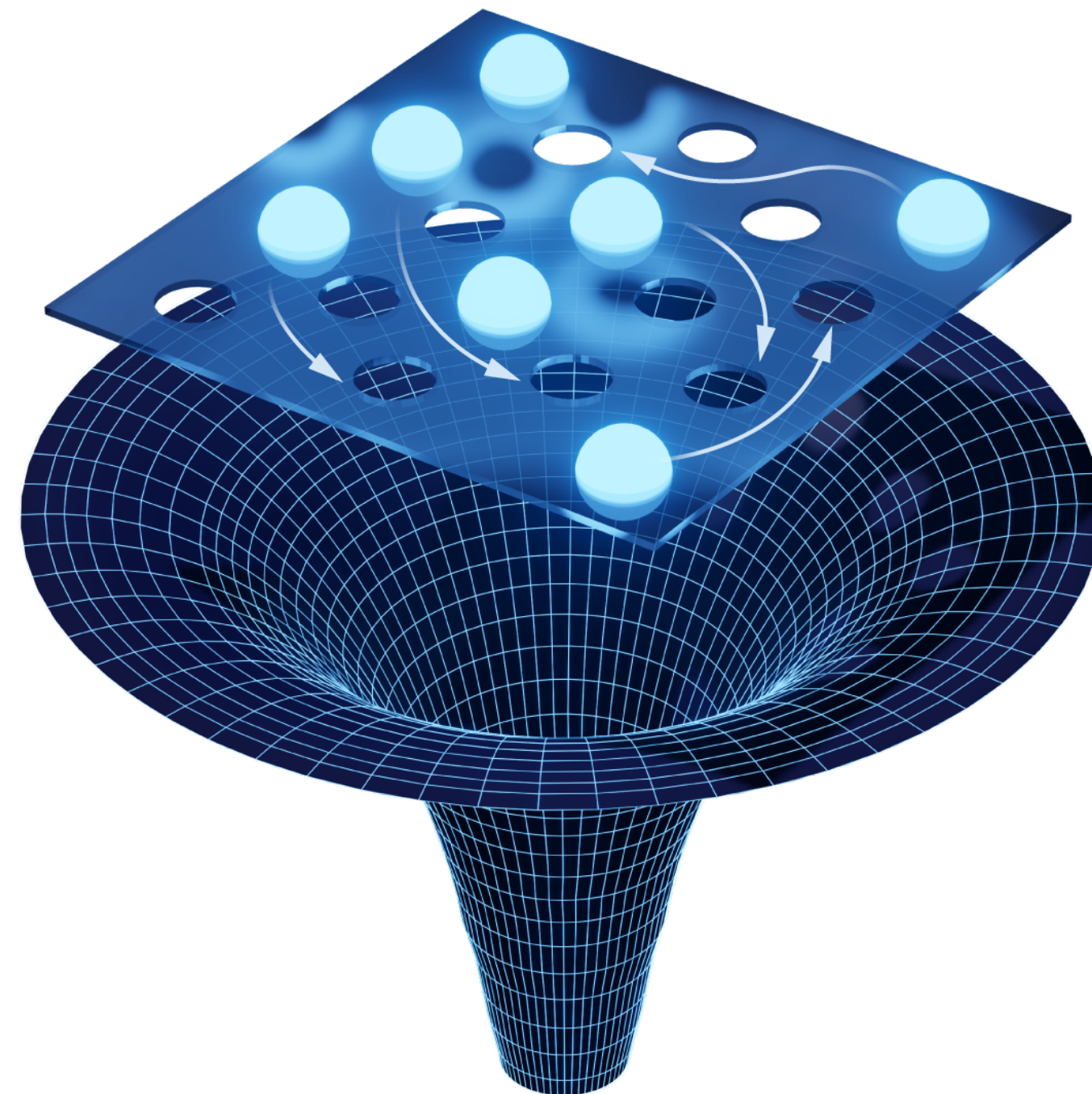
- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.
- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS_2 .

Summary

- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



Summary

- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.
- Linear- T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.

