Interacting electronic topological insulators in 3D: classification and characterization

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Acknoledgments

 

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Discussions: Liang Fu, Adam Nahum, Xie Chen, Evelyn Tang, Tim Hsieh ...

The question

How do interactions change the theory of topological insulators?

Physically motivated:

- Materials with strong spin-orbit and *e*-*e* interaction: Pyrochlore iridates, Kondo insulators, etc.
- Time-reversal & charge-conservation are realistic symmetries

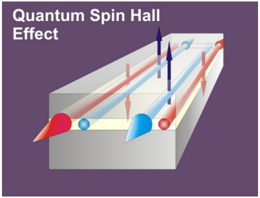
The question

How do interactions change the theory of topological insulators?

This talk:

- Are there new phases beyond band theory?
  - Yes!
- In $3D$: $\mathbb{Z}_2(Free) \to \mathbb{Z}_2^3(Interacting)$
- Characterization? Fingerprints in the lab? ✓

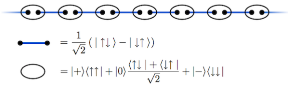
What is an interacting TI?

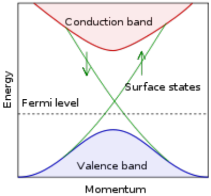


- ~~Band insulator~~ $\rightarrow$ gapped bulk, non-fractional
- Global symmetries: charge conservation $U(1)$ and time-reversal $\mathcal{T}$
- Breaking symmetries $\sim$ atomic insulator

$\rightarrow$ **Symmetry-protected topological** (SPT) state

The notion of SPT





- **SPT**: Symmetry Protected Topological states: gapped & symmetric (Chen, et al. 2011)
- Nontrivial *iff* symmetries are preserved
- Trivial bulk:
  no fractionalization (topological order)
- Also exist in bosons
  classic example: Haldane chain
- Our focus: fermions in $3D$, with $U(1) \rtimes \mathcal{T}$

SPT boundary

- SPT boundary cannot be trivially gapped. Known possibilities include:
  1. Symmetry-breaking
  2. Gapless
  3. Topological order (Vishwanath, Senthil, 2012)
     -even for the famous band TI!
     (CW, Potter, Senthil; Metlitski, Kane, Fisher; Chen, Fidkowski, Vishwanath; Bonderson, Nayak, Qi, arXiv:1306.32**)
- Symmetry-preserving boundary states cannot be realized alone without the SPT bulk (anomalous)

Outline

1. A simple example beyond band theory
2. Classification: from $\mathbb{Z}_2$ to $\mathbb{Z}_2^3$
3. Characterization (in the lab): surface magnets and superconductors
4. Other symmetries: the revised periodic table

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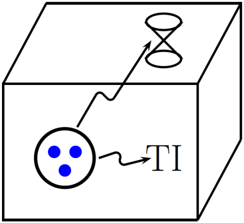
Cluston TI



- Take three-body bound states: charge-3 fermions $F \sim fff$
- Put F into a topological (Fu-Kane-Mele) band
- Break $\mathcal{T}$ on surface: $\sigma_{xy} = \frac{1}{2}(e^*)^2 = \frac{9}{2}$, $\kappa_{xy} = \frac{1}{2}$, $\sigma_{xy} - \kappa_{xy} = 4$
- In strict $2D$ electron systems without topological order, $\sigma_{xy} - \kappa_{xy} = 8n$ (e.g. E_8 state with no hall conductance)
- The cluston TI is different from the band TI! (CW, 2014)

What is it?

Claim: Cluston TI $\cong$ Band TI $\otimes$ A boson SPT

Check: consider Cluston TI $\otimes$ Band TI, surface state:

$$\mathcal{L} = \bar{\psi}\sigma^{\mu}(-i\partial_{\mu} + A_{\mu})\psi + \bar{\Psi}\sigma^{\mu}(-i\partial_{\mu} + 3A_{\mu})\Psi$$

- Pairing gap (breaks $U(1)$):

$$\Delta\mathcal{L} = i\Delta\psi\sigma_y\psi + i\xi(\Delta)^3\Psi\sigma_y\Psi + h.c.$$

- To restore $U(1)$, need to condense vortices

What is it? (cont.)

- Vortex statistics can be calculated

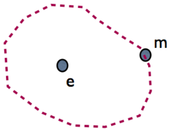
  (Methods developed in CW, Potter, Senthil; Metlitski, Kane, Fisher, 2013)

  $\rightarrow$ strength-1 vortex (π-vortex) is a fermion

- Condensing strength-2 vortex (2π-vortex) $\rightarrow$ $\mathbb{Z}_2$ topological order

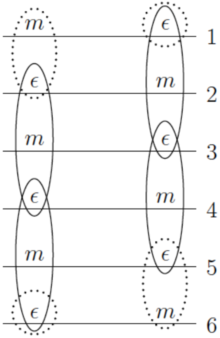
- Excitations: Bogoliubov particle e (fermion, $\mathcal{T}^2 = -1$), un-condensed vortex ϵ (fermion), bound state $m \sim e\epsilon$ (fermion, $\mathcal{T}^2 = -1$)

- $\mathbb{Z}_2$ topological order dubbed $e_f T m_f T$: surface state of a boson SPT with $\mathcal{T}$ only!



Phase of π

Boson SPT: layered construction



- A simple construction ($e_f T m_f T$) with coupled layers of $2D$ $\mathbb{Z}_2$-spin liquids (CW, Senthil, 2013)
- m_i(vison): boson
  ϵ_i(spinon): fermion, $\mathcal{T}^2 = -1$
- Condensing $\epsilon_i m_{i+1} \epsilon_{i+2}$
- Fractional particles in the bulk are confined
- $\epsilon_1 = \tilde{e}_f T$ and $m_1 \epsilon_2 = \tilde{m}_f T$ survive on the boundary $\sim e_f T m_f T$
  (see also Burnell, et al, 2013)

Outline

General TI: EM response

Magneto-electric response (θ-term):

$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2} \vec{E} \cdot \vec{B}$$

- Time-reversal invariance $\rightarrow \theta = 0$ or π (mod 2π)
- If: New TI with $\theta = \pi$
  combine with TBI $\rightarrow$ state with $\theta = 0$

$\rightarrow$ Enough to look at states with $\theta = 0$

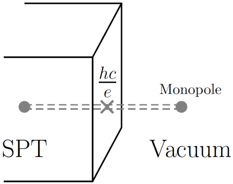
Surface state

- Claim: $\theta = 0 \to$ surface 2π vortex is a boson and can be condensed (prove later)
- Condensing 2π-vortex $\to$ charge quantized in e: particles can be neutralized:

$$\{1, \textit{neutral particles}\} \times \{1, c_\alpha\}$$

- The non-trivial theory is charge-neutral $\to$ must be a bosonic theory!
- Can at most be a charge-neutral boson (spin) SPT: topological paramagnets

From $\theta = 0$ to condensing 2π-vortices



- Surface 2π-vortex can be created through monopole tunneling

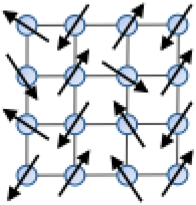
  (see also Metlitski, Kane, Fisher, PRB 2013)
- $\theta = 0 \rightarrow$ monopole carries no charge
- Time-reversal (up to a gauge transform):

$$\mathcal{T} : m \leftrightarrow m^{\dagger}$$

- Monopole must be a boson

  (nontrivial, see CW, Potter, Senthil, 2014 for proof)
- Monopole trivial $\rightarrow hc/e$ trivial (bosonic)

Topological paramagnets in electron systems



$$b \equiv S^- = f_\downarrow^\dagger f_\uparrow$$
$$n_b \equiv S^z = \frac{1}{2}(f_\uparrow^\dagger f_\uparrow - f_\downarrow^\dagger f_\downarrow)$$

- Topological paramagnets in electron systems: Mott insulators $\rightarrow$ spin (boson) systems
- Bosonic SPT with $\mathcal{T}$: classified by $\mathbb{Z}_2^2$
  (Chen, et al, 2011; Vishwanath, Senthil, 2012; Kapustin, 2014; Freed, 2014)
- Total classification $= \mathbb{Z}_2$ (TBI) $\times$ $\mathbb{Z}_2^2$ (Topological paramagnets) $= \mathbb{Z}_2^3$
  (CW, Potter, Senthil, Science 2014)

Another topological paramagnet

The remaining $\mathbb{Z}_2$: another boson SPT with $\mathcal{T}$ only

- Representative surface state: $\mathbb{Z}_2$ topological order, e and m are bosons but $\mathcal{T}^2 = -1$ ($eTmT$)
- Described within group-cohomology
- Cannot be realized in strict $2D$

The result

Topological Insulator	Representative surface state	$\mathcal{T}$-breaking transport signature	$\mathcal{T}$-invariant gapless superconductor
Free fermion TI	Single Dirac cone	$\sigma_{xy} = \frac{\kappa_{xy}}{\kappa_0} = \pm 1/2$	None
Topological paramagnet I $(eTmT)$	$\mathbb{Z}_2$ spin liquid with Kramers doublet spinon(e) and vison(m)	$\sigma_{xy} = \kappa_{xy} = 0$	$N = 8$ Majorana cones
Topological paramagnet II $(e_f m_f)$	$\mathbb{Z}_2$ spin liquid with Fermionic spinon(e) and vison(m)	$\sigma_{xy} = 0; \frac{\kappa_{xy}}{\kappa_0} = \pm 4$	$N = 8$ Majorana cones

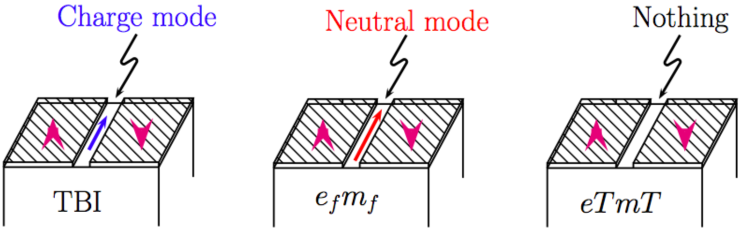
(CW, Potter, Senthil, Science 2014)

- Classification: $\mathbb{Z}_2^3 = \mathbb{Z}_2(\text{free fermion}) \times \mathbb{Z}_2^2(\text{topological paramagnets})$
- What to measure in the lab?
  Surface topological order: difficult
  Symmetry-breaking surface: unique fingerprints!

Outline

Surface magnets: hall transport

Breaking $\mathcal{T}$ (depositing a magnet): gapped without topological order

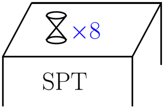


(Vishwanath, Senthil, 2012)

Surface superconductors: Majorana cones

Breaking $U(1)$ (depositing a superconductor): topological orders confined

- TBI: surface superconductors gapped
- $e_f T m_f T$ $\cong$TBI$\oplus$Cluston TI: also gapped
- $eTmT$: surface superconductor gapless! Eight Majorana cones protected by $\mathcal{T}$
  ($\sim$He-3B$\times$8, but bulk can be insulating only in interacting system)



can be seen through ARPES!

Understanding the cones

Argue in reverse: start from eight Majorana cones (four Dirac)

$$H = \sum_{i=1}^{4} \psi_i^\dagger (p_x \sigma^x + p_y \sigma^z) \psi_i$$

with $\mathcal{T}$: $\psi_i \to i\sigma_y \psi_i^\dagger$

- Quadratic gaps forbidden by $\mathcal{T}$ – non-perturbative gap?
- Introduce auxiliary $U(1)$: $\psi_i \to e^{i\theta}\psi_i$
- $H_\Delta = i\Delta(\vec{x})\psi_i\sigma_y\psi_i + h.c.$
  $\to$ breaks $\mathcal{T}$ and $U(1)$, but preserves $\tilde{\mathcal{T}} = \mathcal{T} U_{\pi/2}$

Understanding the cones (cont.)

- Disorder $\Delta(\vec{x})$ ($\langle\Delta(\vec{x})\rangle = 0$) to recover symmetries
- Need to proliferate vortices, but fundamental vortex nontrivial: $\tilde{\mathcal{T}}^2 = -1$
- Condense strength-2 vortex $\rightarrow$ $\mathbb{Z}_2$-ordered $eTmT$ state!
  (related results: Fidkowski, Chen, Vishwanath, 2013; Metlitski, Fidkowski, Chen, Vishwanath, 2014)

Outline

Reduced classifications

Trick of non-perturbative gap: surface states of other TI/TSc in the 10-fold classification (Ryu, et al; Kitaev, 2009)

- States with trivial defects: trivial under interaction $\rightarrow$ reduction of classification
- Example: $\mathcal{T}$-invariant superconductors: $\mathbb{Z}$-classification in free fermions
- Strong interaction: $\mathbb{Z} \rightarrow \mathbb{Z}_{16}$
  (Fidkowski, Chen, Vishwanath, 2013; see also Cenke Xu's Talk)
- Non-perturbative gap approach: intuitive understanding & generalization to other symmetries

Revised periodic table in 3D

- Boson SPT gives many phases beyond free fermions
- Apply monopole & surface state argument: complete classification when symmetry contains a normal $U(1)$ subgroup
  Example: TSc with $\mathcal{T}$ and S_z conservation: $\mathbb{Z}_8 \times \mathbb{Z}_2$

Symmetry class	Reduction of free fermion states	Distinct boson SPT	Complete classification
$U(1)$ only (A)	0	0	0
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (AII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^3$
$U(1) \rtimes \mathbb{Z}_2^T$ with $\mathcal{T}^2 = 1$ (AI)	0	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$U(1) \times \mathbb{Z}_2^T$ (AIII)	$\mathbb{Z} \to \mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_8 \times \mathbb{Z}_2$
$U(1) \rtimes (\mathbb{Z}_2^T \times \mathbb{Z}_2^C)$ (CII)	$\mathbb{Z}_2 \to \mathbb{Z}_2$	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^5$
$(U(1) \rtimes \mathbb{Z}_2^T) \times SU(2)$	0	$\mathbb{Z}_2^4$	$\mathbb{Z}_2^4$
$\mathbb{Z}_2^T$ with $\mathcal{T}^2 = -1$ (DIII)	$\mathbb{Z} \to \mathbb{Z}_{16}$	0	Z_{16} (?)
$SU(2) \times \mathbb{Z}_2^T$ (CI)	$\mathbb{Z} \to \mathbb{Z}_4$	$\mathbb{Z}_2$	$Z_4 \times Z_2$ (?)

(CW, Senthil, 2014)

Symmetry-enforced gaplessness

A new paradigm(?) in 3D: gapped symmetric surfaces, with intrinsic topological order

- **A counter example**: $SU(2)\times\mathcal{T}$-invariant topological superconductor, with one Dirac cone per spin on surface

$$H=\sum_{\alpha=\uparrow,\downarrow}\psi^{\dagger}_{\alpha}(p_x\sigma^x+p_y\sigma^z)\psi_{\alpha}$$

(Schnyder, Ryu, Ludwig, 2009)

- cannot be symmetrically gapped, even with topological order

Symmetry-enforced gaplessness: proof

- Suppose the surface could be gapped: $\{1, X, ...\}$
- $SU(2)$ has no projective representation
  $\rightarrow X$ carries half-integer spin
- Bind electrons to X to produce spin-singlet object $\tilde{X}$
- Rewrite surface theory as $\{1, \tilde{X}, ...\}$: spin singlet
  $\rightarrow$ bosonic theory $\rightarrow$ bulk state equivalent to a boson SPT
- However, tow copies of the state gives a boson SPT ($eTmT$)
  $\rightarrow$ the state itself cannot be a boson SPT

Conclusion

Further thoughts:

- Material realization? Frustrated spin-1 magnets may have a chance (ongoing work with Adam Nahum, T. Senthil)
- Implications on other issues: spin liquids (ongoing work with T. Senthil), standard model (Cenke Xu's talk)...

Thank you!