Mini-School on Nonlinear Equations
Program

Saturday, December 3

9:00-10:30am Clifford Taubes, Harvard University

Title: "Compactness theorems in gauge theories"

Abstract: The algebraic counting of solutions to the self-dual Yang Mills equation and the Seiberg-Witten equation leads to invariants of smooth 3 and 4-dimensional manifolds; and invariants of surfaces in 4-manifolds and invariants of knots in 3-manifolds. The counting requires that the set of objects to be counted be finite (or compact in suitable topology). There are other gauge theory equations (generalized Seiberg-Witten/self-dual equations) where the solution space is, in no sense, finite or compact. This being the case, any hypothetical application of these more general equations will likely require a thorough understanding of the behavior of non-convergent sequences of solutions. Of particular importance are theorems that characterize non-convergence geometrically. The two lectures will give an introduction to some of the analysis that has been used to characterize non-convergent sequences of these more general gauge theory equations.

10:30-10:45pm Break

10:45am-12:15pm Valentino Tosatti, Northwestern University

Title: "Complex Monge-Amp\`ere Equations"

Abstract: I will give an introduction to the study of complex Monge-Amp\`ere equations on compact K\"ahler manifolds, going over the analytic techniques that are used to solve such equations, and their geometric applications.

12:15-1:45pm Lunch

1:45-3:15pm Pengfei Guan, McGill University

Title: "Monge-Amp\`ere type equations and related geometric problems"

Abstract: The two lectures cover recent developments on some longstanding problems in geometry. The first lecture will start from classical solution to the Minkowski problem (Nirenberg, Cheng-Yau and Pogorelov) and associated Monge-Amp\`ere equation. We then move to the problem of prescribing area measure in convex geometry (the Christofell-Minkowski problem), dealing with fundamental issue of convexity in nonlinear PDE, via constant rank theorem developed based on ideas of Caffarelli-Friedman and Yau in 1980s. The second lecture will concentrate on recently obtained curvature estimates for nonlinear geometric PDEs, solving the Alexandrov problem of prescribing curvature measures as an example. We will also discuss some open problems.

3:15-3:30pm Break

3:30-5:00PM Jared Speck, MIT

Title: "Finite-Time Degeneration of Hyperbolicity Without Blowup for Solutions to Quasilinear Wave Equations"

Abstract: In many physical models, the fundamental evolution equations are quasilinear and wave-like, including Einstein's equations, the equations of irrotational compressible fluid mechanics, elasticity, and nonlinear electromagnetism. For many of these equations, there are results on the breakdown of smooth solutions due to the formation of a singularity, that is, because the solution or one of its higher derivatives blows up infinite time. However, there is another kind of degeneracy that can occur in solutions to certain quasilinear equations, which can potentially serve as an obstacle to continuing the solution: a degeneration of the hyperbolicity of the equation corresponding, roughly, to the finite-time vanishing of a speed of propagation. In a recent article, I studied model quasilinear wave equations and proved, for an open set of initial data, the first result of this type in more than one spatial dimension. Interestingly, the solution variable does not experience any singularity at the degeneration points, though many things can in principle go wrong at such points, depending on the structure of the nonlinearities. In this mini-course, we will discuss the proof in detail. I will then discuss open problems, which are bountiful since there are no other results in this direction.

Sunday, December 4

9:00-10:30AM Clifford Taubes, Harvard University

Title: "Compactness theorems in gauge theories"

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