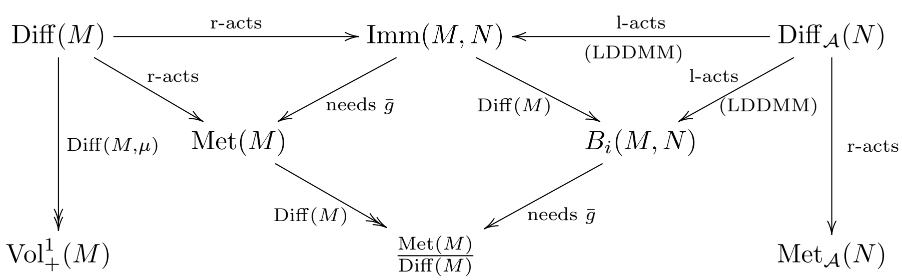
Abstract. On the space *Emb* of all smooth embeddings (more general: immersions) from a compact manifold M into a Riemannian manifold $(N, \bar{g})$ act the Lie group of all diffeomorphisms of M from the right, and various groups of diffeomorphisms of N from the left. Quotiening out the right action leads to a prime example of "shape space", also called the "differentiable Chow variety" or the "nonlinear Grassmannian", consisting of all submanifolds of N of type M. Invariant Riemannian metrics on *Emb* lead to Riemannian metrics on shape space, whose geodesic distances can be used to differentiate between shapes, and whose curvatures affect statistics of shapes. The left action, via right invariant Riemannian metrics on the diffeomorphism groups of N, also induces various metrics on shape spaces which have found many applications from paelontology to computational anatomy.

In this overview talk I will illustrate many aspects of this circle of ideas; the results came out of collaboration with David Mumford. I will always use the space of differentiable immersed plane curves as the most basic example.

The following diagram summarizes all spaces and actions:



M compact , N manifold	
$\mathrm{Met}(N) = \Gamma(S^2_+ T^* N)$	space of all Riemann metrics on N
$\bar{g}$	one Riemann metric on N
$\mathrm{Diff}(M)$	Lie group of all diffeos on compact mf M
$\mathrm{Diff}_{\mathcal{A}}(N),\ \mathcal{A} \in \{H^\infty, \mathcal{S}, c\}$	Lie group of diffeos of decay $\mathcal{A}$ to Id_N
$\mathrm{Imm}(M, N)$	mf of all immersions $M \to N$
$B_i(M, N) = \mathrm{Imm}/\mathrm{Diff}(M)$	shape space
$\mathrm{Vol}^1_+(M) \subset \Gamma(\mathrm{vol}(M))$	space of positive smooth probability densities

Here is the simple situation of smooth immersed plane curves: