Noncommutative Analysis, Computational Complexity, and Quantum Information

October 16-18, 2019

Wednesday, October 16 - Tutorial Lecture Day

Time	Speaker	Title/Abstract
8:30 - 9:00am	**Breakfast**	
9:00 - 9:45am	Ankit Garg & Rafael Oliveira	**Part I** **Title:** Invariant theory, noncommutative optimization and applications **Abstract:** In the past few years, a lot of work has been done in designing algorithms for certain optimization problems arising from invariant theory. These optimization problems weave a myriad of connections within several areas of math, physics and computer science and have found several applications in non-commutative algebra, computational complexity, quantum information, invariant theory, functional analysis, the theory of tensors, operator theory and combinatorial geometry. The first part of the tutorial will give an overview of some of these connections especially the ones relevant to the topics of the workshop, e.g. identity testing of non-commutative rational expressions and quantum marginal problems. The second part will focus on various algorithms developed for these problems and their analyses. We will also survey open problems in the area.
9:45 - 10:00am	Discussion	
10:00 - 10:45am	Ankit Garg & Rafael Oliveira	**Part II**
10:45 - 11:00am	Discussion	
11:00 - 11:30am	**Break**	
11:30am - 12:15pm	Igor Klep	**Part I** **Title:** Noncommutative Real Algebraic Geometry: A Survey **Abstract:** The last decade has seen the development of a substantial noncommutative (in a free algebra) real and complex algebraic geometry. The aim of the subject is to develop a systematic theory of equations and inequalities for nc (noncommutative) polynomials or rational functions of matrix variables and these talks will survey the main techniques and results. The plan at this moment is to start with nc positivstellensatze and nullstellensatz, then go to Linear Matrix Inequalities and nc realizations and some of their consequences.
12:15 - 12:30pm	Discussion	
12:30 - 1:30pm	**Lunch**	

1:30 - 2:15pm	Igor Klep	**Part II**
2:15 - 2:30pm	Discussion	
2:30 - 3:15pm	Aram Harrow	**Title:** Sum-of-squares hierarchies and quantum information **Abstract:** I will review three ways in which the sum-of-squares (SoS) and non-commutative SoS (ncSoS) hierarchies can be applied to problems in quantum information. 1. Optimizing over separable states is equivalent to determining whether a state is entangled or not as well as to apparently non-quantum problems such as calculating the L2->L4 norm of a matrix. The complexity of crude approximations to this problem appears to be somewhere between P and NP, and has connections to the unique games problems. The SoS algorithm for this problem can be analyzed using tools of quantum information theory, such as the monogamy of entanglement, and has nontrivial approximation guarantees in some cases. 2. Finding the ground state of a local Hamiltonian is QMA-complete, which is the quantum analogue of being NP-complete. In practice variational methods are often used to yield upper bounds on the true ground state energy. SoS, by contrast, yields lower bounds. More general variants can be used to estimate quantities such as free energy. Here relatively few rigorous results are known and I will discuss the state of knowledge. 3. The ncSoS hierarchy can be used to estimate the quantum value of a nonlocal game in which a referee interacts with a team of two or more players who share entanglement. This problem is in general undecidable but there are many cases in which we can understand the performance of the ncSoS hierarchy.
3:15 - 3:30pm	Discussion	
3:30 - 4:00pm	**Tea**	
4:00 - 5:00pm	**Colloquium:** Aram Harrow	**Title:** Monogamy of entanglement and convex geometry **Abstract:** The SoS (sum of squares) hierarchy is a flexible algorithm that can be used to optimize polynomials and to test whether a quantum state is entangled or separable. (Remarkably, these two problems are nearly isomorphic.) These questions lie at the boundary of P, NP and the unique games conjecture, but it is in general open how well the SoS algorithm performs. I will discuss how ideas from quantum information (the "monogamy" property of entanglement) can be used to understand this algorithm. Then I will describe an alternate algorithm that relies on apparently different tools from convex geometry that achieves similar performance. This is an example of a series of remarkable parallels between SoS algorithms and simpler algorithms that exhaustively search over carefully chosen sets. Finally, I will describe known limitations on SoS algorithms for these problems.

Thursday, October 17

Time	Speaker	Title/Abstract
8:30 - 9:00am	**Breakfast**	
9:00 - 9:45am	Boaz Barak	**Title:** The Bayesian view of optimization and the quantum separability problem
9:45 - 10:00am	Discussion	
10:00 - 10:45am	Vern Paulsen	**Title:** Algebras and Synchronous Games **Abstract:** It is known that some games can be won with higher probability if the players are allowed to use random outcomes produced by entanglement. This is because the set of conditional probability densities that can be generated in this fashion is strictly larger than the densities that can be produced classically. But currently there are several mathematical models for describing these sets of ``quantum conditional densities"" and we have only recently learned that some of the models are different. The largest such set turns out to be an intersection of a decreasing sequence of spectrahedra. In this talk I will focus on perfect strategies, i.e., random strategies produce a losing outcome with probability 0. Each synchronous game has an affiliated noncommutative *-algebra whose properties determine whether or not the game has a perfect quantum strategy. For some games we know that the existence of a perfect strategy can be reduced to a Grobner basis problem for these algebras.
10:45 - 11:00am	Discussion	
11:00 - 11:30am	**Break**	
11:30am - 12:15pm	Adam Bene Watts	**Title:** An Algebraic Framework for XOR Games **Abstract:** XOR games are nonlocal games where players respond to questions with a single bit, and the verifier accepts or rejects responses based on the overall parity of the players' responses. We consider the question of deciding whether these games have a perfect quantum (commuting operator) strategy. The ncSoS algorithm puts this problem in the class coRE; that is, the ncSoS algorithm is guaranteed to terminate in the no case (with no guarantee on run time) and may run forever on a yes instance. However, structure specific to XOR games suggests a more tailored algorithm may be able to succeed in cases where ncSoS struggles. In this talk I will show the question of determining existence of a perfect commuting operator strategy for XOR games reduces to an instance of the subgroup membership problem on a Coxeter group. I'll discuss some special cases of games for which the associated subgroup membership problem is decidable in polynomial time, including games where ncSoS takes doubly exponential time to conclude the game does not have a commuting operator strategy. I'll also discuss the open question of

		whether the subgroup membership problems associated with XOR games are in general decidable.
12:15 - 12:30pm	Discussion	
12:30 - 2:00pm	**Lunch**	
2:00 - 2:45pm	Scott McCullough	**Title:** Matrix inequalities, noncommutative convexity, completely positive maps and linearizations. **Abstract:** The theory of matrix inequalities is a freely noncommutative analog of semialgebraic geometry. Issues of convexity for these inequalities are motivated by applications and intersect with many areas of mathematics, including the theory of completely positive maps, matrix convexity and linearizations. As time permits, we will discuss spectrahedral inclusions, unital cp maps, and semidefnite programs; a notion of convex sets related to trace preserving cp maps; noncommutative rational mappings as nonlinear cp maps; and convexity for noncommutative rational maps and linearizations.
2:45 - 3:00pm	Discussion	
3:00 - 3:30pm	**Tea**	
3:30 - 4:15pm	Ion Nechita	**Title**: Compatibility of quantum measurements and inclusion of free spectrahedra **Abstract:** One of the defining properties of quantum mechanics is the existence of incompatible observables, of which the observables of position and momentum are a well-known example. In this talk, we will connect the problem of determining whether a given set of quantum measurements is compatible to the inclusion of free spectrahedra. We show how results from algebraic convexity can be used to quantify the degree of incompatibility of binary quantum measurements. In particular, this new connection will allow to completely characterize the case in which the dimension of the quantum system is exponential in the number of measurements. Vice-versa, results from quantum information theory, such as quantum cloning, yield bounds for the inclusion constants of a class of free spectrahedra obtained as direct sums of simplices.
4:15pm	Jason Altschuler	**Title:** Simple, practical, nearly-optimal algorithms for matrix scaling and optimal transport **Abstract:** Matrix Scaling is a fundamental problem in scientific computing used pervasively as a pre-conditioning subroutine. The practitioners' algorithm of choice for Matrix Scaling has long been the Sinkhorn scaling algorithm, ever since its introduction in the 1960s. However, despite the widespread use of this algorithm and its great empirical performance, its theoretical guarantees were not fully understood. In this talk, I will give a short, simple proof that Sinkhorn scaling converges in near-linear time in the matrix size. I will then describe how this improvement for Matrix Scaling yields a simple,

		practical, near-linear-time approximation algorithm for Optimal Transport—an increasingly popular tool in machine learning, statistics, and computer vision.
	General Discussion	Everyone is urged to make comments on their fields vs. others, how areas might interact.

Friday, October 18

Time	Speaker	Title/Abstract
8:30 - 9:00am	**Breakfast**	
9:00 - 9:45am	David Gosset	**Title:** Approximation algorithms for quantum many-body problems **Abstract:** Computing the ground energy of a quantum many-body system can be an intractable problem. A common approach to obtain an approximate solution is to variationally optimize over a subset of quantum states that can be efficiently represented using a classical computer. In this talk I will discuss the worst-case performance of approximate optimization algorithms for quantum spin and fermionic systems with two-body interactions, including mean-field theory (optimization over product states) and Hartree-Fock theory (Slater determinants or fermionic Gaussian states). I will describe the performance of efficient algorithms based on semidefinite programming relaxation methods, as well as bounds on the best possible energy attained by quantum states in these variational families. This talk is based on joint work with S.Bravyi, R. Koenig, and K. Temme (arXiv: 1808.01734).
9:45 - 10:00am	Discussion	
10:00 - 11:00am	Suvrit Sra	**Title:** Some non-convex optimization problems through a geometric lens **Abstract:** In this talk I wish to draw your attention to a subclass of tractable nonconvex problems. In particular, I will present a rich subclass of nonconvex problems that can be solved to global optimality by leveraging geometry. The key concept that I will talk about is geodesic convexity, which generalizes the usual vector-space notion of convexity to nonlinear spaces such as some manifolds and metric spaces. My aim is to outline how geometric thinking leads to improved models or insights for notable problems such as large-scale PCA, metric learning, Gaussian mixture models, the Brascamp-Lieb constant, Wasserstein Barycenters, among others. I will outline both theoretical

		and practical aspects, while raising some open problems.
10:45 - 11:00am	Discussion	
11:00 - 11:30am	**Break**	
11:30am - 12:15pm	Jurij Volčič	**Title:** Matricial singularities of noncommutative polynomials **Abstract:** Given a noncommutative polynomial f, its free locus is the family of determinant=0 hypersurfaces consisting of matrix tuples X such that f(X) is a singular matrix. This is a noncommutative analog of a hypersurface in (real) algebraic geometry. They appear prominently as boundaries of free semialgebraic sets, singularities of noncommutative rational functions, and the geometric counterpart of factorization in a free algebra. This talk discusses main features of free loci. The central ones are the correspondence between components of the free locus of f and irreducible factors of f, and a Nullstellensatz: a noncommutative polynomial g attains singular values wherever f attains singular values if and only if irreducible factors of f are essentially factors of g. The talk also touches upon linearization methods and representation/invariant theory used in proving these results.
12:15 - 12:30pm	Discussion	
12:30 - 2:00pm	**Lunch**	
2:00 - 2:45pm	Salma Kuhlmann	**Title:** Fields of Generalized Power Series (the mysteries and obstacles around formal summability). **Abstract:** Fields of generalized (formal) power series (with exponents in totally ordered *abelian* groups) are obtained via formal constructions, widely generalizing those of the classical Laurent or Puiseux series. They are natural (pseudo) completions of fields of fractions of group rings. They play a central role in many areas of mathematics, logic, and theoretical computer science, e.g. in commutative algebra, (real) algebraic geometry, asymptotic analysis, asymptotic differential algebra, dynamical systems, model theory and automata theory, to mention just a few. A crucial issue (and often a major tool or obstacle) concerns the formal summability of families of formal series. We will focus on this aspect and illustrate by examples how it is handled while formally defining algebraic, analytic, difference and differential operators on these objects. Although the objects under consideration are commutative, we hope that our contribution could provide thoughts for non-commutative versions.
2:45 - 3:00pm	Discussion	
3:00 - 3:45pm	Victor Vinnikov	**Title:** Germs of nc analytic functions at a matrix point **Abstract:** I will discuss the ring of germs of nc functions at an (irreducible) matrix point. Two key results are that this ring coincides with the (algebraic) completion of the free algebra and that except for the scalar case, it (and even its subring of germs of analytic nc

		functions) always contains nilpotents. The main tools are a Hermite type interpolation theorem for the free algebra and generalized nc power series (Taylor--Taylor series). The talk is based on joint work with I. Klep and Ju. Volcic.
3:45 - 4:00pm	Discussion	
4:00 - 5:00pm	**Tea & Discussion**	