

Immersions of manifolds and homotopy theory

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The interface between the study of the topology of differentiable manifolds and algebraic topology has been one of the richest areas of work in topology since the 1950's. In this talk I will focus on one aspect of that interface: the problem of studying embeddings and immersions of manifolds using homotopy theoretic techniques. I will discuss the history of this problem, going back to the pioneering work of Whitney, Thom, Pontrjagin, Wu, Smale, Hirsch, and others. I will discuss the historical applications of this homotopy theoretic perspective, going back to Smale's eversion of the 2-sphere in 3-space. I will then focus on the problems of finding the smallest dimension Euclidean space into which every n -manifold embeds or immerses. The embedding question is still very much unsolved, and the immersion question was solved in the 1980's. I will discuss the homotopy theoretic techniques involved in the solution of this problem, and contributions in the 60's, 70's and 80's of Massey, Brown, Peterson, and myself. I will also discuss questions regarding the best embedding and immersion dimensions of specific manifolds, such as projective spaces. Finally, I will end by discussing more modern approaches to studying spaces of embeddings due to Goodwillie, Weiss, and others. This talk will be geared toward a general mathematical audience.