LECTURE SERIES

Mathematical Science Literature

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9:00am ET- *Virtually*

"Quantum topology and new types of modularity"

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The talk concerns two fundamental themes of modern 3-dimensional topology and their unexpected connection with a theme coming from number theory. A deep insight of William Thurston in the mid-1970s is that the vast majority of complements of knots in the 3-sphere, or more generally of 3-manifolds, have a unique metric structure as hyperbolic manifolds of constant curvature -1, so that 3-dimensional topology is in some sense not really a branch of topology at all, but of differential geometry. In a different direction, the work of Vaughan Jones and Ed Witten in the late 1980s gave rise to the field of Quantum Topology, in which new types of invariants of knot complements and 3-manifolds are introduced that have their origins in ideas coming from quantum field theory. These two themes then became linked by Kashaev's famous Volume Conjecture, now some 25 years old, which says that the Kashaev invariant _N of a hyperbolic knot K (this is a quantum invariant defined for each positive integer N and whose values are algebraic numbers) grows exponentially as N tends to infinity with an exponent proportional to the hyperbolic volume of the knot complement. About 10 years ago, I was led by numerical experiments to the discovery that Kashaev's invariant could be upgraded to an invariant having rational numbers as its argument (with the original invariant being the value at 1/N) and that the Volume Conjecture then became part of a bigger story saying that the new invariant has some sort of strange transformation property under the action x -> (ax+b)/(cx+d) of the modular group SL(2,Z) on the argument. This turned out to be only the beginning of a fascinating and multi-faceted story relating quantum invariants, q-series, modularity, and many other topics. In the talk, which is intended for a general mathematical audience, I would like to recount some parts of this story, which is joint work with Stavros Garoufalidis (and of course involving contributions from many other authors). The "new types of modularity" in the title refer to a specific byproduct of these investigations, namely that there is a generalization of the classical notion of holomorphic modular form – which plays an absolutely central role in modern number theory – to a new class of holomorphic functions in the upper half-plane that no longer satisfy a transformation law under the action of the modular group, but a weaker extendability property instead. This new class, called "holomorphic quantum modular forms", turns out to contain many other functions of a more number-theoretical nature as well as the original examples coming from quantum invariants.

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